

S.B. Arts and

K.C.P Science

College VIJAYAPUR

Name: Bhagyashree - Samagond

Class: BSc 1st sem

UUCMS NO: WSKM2350243

Subject: Maths Seminar

DEPARTMENT OF MATHEMATICS

SEMINAR ON

LAGRANGE'S MEAN VALUE THEOREM

Lagrange's Mean Value Theorem

Let $f(x)$ be a function that satisfies the following two conditions.

- (i) $f(x)$ is continuous on $[a, b]$
- (ii) $f(x)$ is differentiable on (a, b) then there exist at least one point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Proof = Define a function $g(x) = f(x) - Ax \quad \forall x \in [a, b]$
Where A is a constant to be determined such that

$$g(a) = g(b)$$

$$f(a) - Aa = f(b) - Ab$$

$$Ab - Aa = f(b) - f(a)$$

$$A(b - a) = f(b) - f(a)$$

$$A = \frac{f(b) - f(a)}{b - a} \quad \text{--- (1)}$$

Since $f(x)$ is continuous on $[a, b]$ & differentiable on (a, b)

$g(x)$ is also continuous in $[a, b]$ & differentiable (a, b)
also $g(a) = g(b)$

hence $g(x)$ satisfies three conditions of Rolle's theorem $\therefore$ there exist at least one point $c \in (a, b)$

such that $g'(c) = 0$

$$g'(x) = f'(x) - A$$

$$g'(c) = f'(c) - A$$

$$f'(c) - A = 0$$

$$A = f'(c) \quad \text{--- (2)}$$

from equation (1) & (2)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Example

1) Verify Lagrange's Mean value theorem examples

$$f(x) = (x+1)(x+2) \text{ in } [1, 2]$$

$$\begin{aligned} \text{let } f(x) &= x^2 + 2x + x + 2 \\ &= x^2 + 3x + 2 \end{aligned}$$

i) $f(x) = x^2 + 3x + 2$ is a polynomial in x it is continuous for every value of x

$\Rightarrow f(x)$ is continuous on $[1, 2]$

ii) Derivative $f'(x) = 2x + 3$ exist in $(1, 2)$

$\Rightarrow f(x)$ is differentiable on $(1, 2)$

hence $f(x)$ satisfies two conditions of Lagrange's Mean value theorem

$\therefore \exists$ at least one point $c \in (a, b)$

$$\text{such that } f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f(b) = f(2) = 4 + 6 + 2 = 12$$

$$f(a) = f(1) = 1 + 3 + 2 = 6$$

$$f'(c) = 2c + 3$$

$$2c+3 = \frac{12-6}{2-1}$$

$$2c+3 = \frac{6}{1} = 6$$

$$2c+3 = 6$$

$$2c = 3$$

$$c = \frac{3}{2} \in (1, 2)$$

2) Verify Lagrange's Mean Value theorem.

$$f(x) = (x-1)(x-2)(x-3) \text{ in } [0, 4]$$

$$\begin{aligned} \text{let } f(x) &= (x^2 - 2x - x + 2)(x-3) \\ &= (x^2 - 3x + 2)(x-3) \\ &= x^3 - 3x^2 - 3x^2 + 9x + 2x - 6 \end{aligned}$$

$$f(x) = x^3 - 6x^2 + 11x - 6$$

(i) $f(x) = x^3 - 6x^2 + 11x - 6$ is polynomial in x it is continuous for every value of x

$\Rightarrow f(x)$ is continuous on $[0, 4]$

(ii) Derivative $f'(x) = 3x^2 - 12x + 11$ exist in $(1, 2)$.

$\Rightarrow f(x)$ is differentiable on $(1, 2)$

hence $f(x)$ satisfies two conditions of Lagrange's theorem

$\therefore$ $\exists$ at least on point $c \in (a, b)$

$$\text{such that } f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\begin{aligned} f(b) &= f(4) = (4)^3 - 6(4)^2 + 11(4) - 6 \\ &= 64 - 96 + 44 - 6 \\ &= 108 - 102 \\ &= 6 \end{aligned}$$

$$f(a) = f(0) = 0 - 6(0) + 11(0) - 6$$

$$= -6$$

$$f'(c) = 3c^2 - 12c + 11$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$3c^2 - 12c + 11 = \frac{6 - (-6)}{4 - 0}$$

$$3c^2 - 12c + 11 = \frac{12}{4} = 3$$

$$3c^2 - 12c + 8 = 0$$

$$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Put } a = 3 \quad b = -12 \quad c = 8$$

$$c = \frac{-(-12) \pm \sqrt{144 - 4 \times 3 \times 8}}{2 \times 3}$$

$$= \frac{12 \pm \sqrt{48}}{6}$$

$$= \frac{12 \pm \sqrt{16 \times 3}}{6}$$

$$\boxed{c = \frac{12 \pm 4\sqrt{3}}{6}} //$$

or

$$c = \frac{\cancel{4}^2 (3 \pm \sqrt{3})}{\cancel{6}_3}$$

$$\boxed{c = \frac{6 \pm 2\sqrt{3}}{3}} //$$

BLDEA'S

S B Arts and

KCP Science

College Vijayapur

Seminar on : Numerical Analysis

Name : Bhagyanidhi Budihal

Sub : Mathematics (II) (B div)

Class : BSc VI sem

VUCMS NO: U15KM21SD339

Seen

1) Find a real root of Eqn $x^3 - 2x - 5 = 0$ by the method of false position correct to 3 decimal places

let $f(x) = x^3 - 2x - 5$ then

$$f(0) = -5, \quad f(1) = -6, \quad f(2) = -1, \quad f(3) = 16$$

$\therefore$ the root lies in $(2, 3)$ & it may be observed that value of $f(x)$ at $x=2$ being -1 is nearer to zero compared to $f(3)=16$, so that we expect root lies in right neighbourhood of 2

$$\text{Now } f(2.1) = 0.061 > 0$$

$\therefore$ the roots lies b/w 2 & 2.1

I appro-

$$\text{here } a = 2 \quad b = 2.1$$

$$f(a) = -1 \quad f(b) = 0.061$$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2(0.061) - 2.1(-1)}{0.061 + 1} = 2.0942$$

Now,

$$f(2.0942) = 9.18447 - 4.1884 - 5 = -0.0039 < 0$$

$\therefore$ the root lies b/w 2.0942 & 2.1

II appro-

$$x_2 = \frac{2.0942(0.061) - 2.1(-0.0039)}{0.061 + 0.0039} = 2.094$$

Comparing x_1 & x_2 we observe that the values are same upto 3 decimal places

hence approximate root is 2.094

2) Solve by Gauss-Jordan Method

$$2x + 5y + 7z = 52$$

$$2x + y - z = 0$$

$$x + y + z = 9$$

The given system of eqn can be written in the form of $AX = B$

$$\begin{bmatrix} 2 & 5 & 7 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 52 \\ 0 \\ 9 \end{bmatrix}$$

Consider the augmented matrix $[A:B]$

$$[A:B] = \left[\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \\ 1 & 1 & 1 & 9 \end{array} \right]$$

$$R_1 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \\ 2 & 5 & 7 & 52 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1 \quad \& \quad R_3 \rightarrow R_3 - 2R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array} \right]$$

$$R_1 \rightarrow R_1 + R_2 \quad \& \quad R_3 \rightarrow R_3 + 3R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array} \right]$$

$$R_3 \rightarrow (-1/4) R_3$$

$$[A:B] \sim \begin{bmatrix} 1 & 0 & 2 & : & -9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

$$R_1 \rightarrow R_1 + 2R_3$$

$$R_2 \rightarrow R_2 + 3R_3$$

$$[A:B] \sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & -1 & 0 & : & -3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

$$x=1, y=3, z=5$$

Hence the required solution is

$$x=2, y=3, z=5.$$

3) Find the Polynomial by using Newton forward interpolation formula.

$$\begin{array}{cccc} x: & 0 & 1 & 2 & 3 \\ f(x): & 1 & 2 & 1 & 10 \end{array}$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
1	2	1		
2	1	-1	-2	
3	10	9	10	12

34) write Newton forward interpolation formula

$$f(x) = f(a) + u \cdot \frac{\Delta f(a)}{1!} + u(u-1) \frac{\Delta^2 f(a)}{2!} + \dots + u(u-1)(u-2)\dots(u-n+1) \frac{\Delta^n f(a)}{n!}$$

here, $h=1$

$$x = a + hu$$

$$u = \frac{x-a}{h} \Rightarrow \frac{x-0}{1} = x \quad [a=0]$$

$$\boxed{u=x}$$

$$f(x) = f(0) + \frac{x \Delta f(0)}{1} + \frac{x(x-1) \Delta^2 f(0)}{2} + \frac{x(x-1)(x-2) \Delta^3 f(0)}{6}$$

$$= 1 + x(1) + \frac{x(x-1)(-2)}{2} + \frac{x(x-1)(x-2)(\frac{2}{2})}{6}$$

$$= 1 + x - (x^2 - x) + x(x^2 - 2x - x + 2)2$$

$$= 1 + x - x^2 + x + 2x(x^2 - 3x + 2)$$

$$= 1 + x - x^2 + x + 2x^3 - 6x^2 + 4x$$

$$f(x) = 2x^3 - 5x^2 + 6x + 1$$

which is required polynomial.

47 find $f'(1.5)$ from given table

x :	0.0	0.5	1.0	1.5	2.0
y :	0.3989	0.3521	0.2420	0.1295	0.0540

consider diff table

x	y	$\nabla f(x_n)$	$\nabla^2 f(x_n)$	$\nabla^3 f(x_n)$	$\nabla^4 f(x_n)$
0.0	0.3989				
0.5	0.3521	-0.0468			
1.0	0.2420	-0.1101	-0.0633		
1.5	0.1295	-0.1125	-0.0024	0.0609	
2.0	0.0540	-0.0755	0.037	0.0394	-0.0215

here $h=0.5$ $x=1.5$ $x_n=1.5$ $P = \frac{1.5-1.5}{0.5} = 0$

$$f'(x_n) = \frac{1}{h} \left[\nabla f(x_n) + \frac{1}{2} \nabla^2 f(x_n) + \frac{1}{3} \nabla^3 f(x_n) \right]$$

$$f'(1.5) = \frac{1}{0.5} \left[-0.1125 + \frac{1}{2} (-0.0024) + \frac{1}{3} (0.0609) \right]$$

$$= \frac{1}{0.5} [-0.1125 - 0.0012 + 0.0203]$$

$$= \frac{1}{0.5} [-0.0934]$$

$$f'(1.5) = -0.1868$$

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Name: Bhagyashree - Samagond

Class: BSc 1st Sem

UUCMS NO: WSKM23502U3

Subject: Maths Seminar

DEPARTMENT OF MATHEMATICS

SEMINAR ON

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$$A = f'(c) \quad \text{--- (2)}$$

from equation (1) & (2)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

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hence $f(x)$ satisfies two conditions of Lagrange's Mean value theorem

$\therefore \exists$ at least one point $c \in (a, b)$
such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

$$f(b) = f(2) = 4 + 6 + 2 = 12$$

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2) Verify Lagrange's Mean Value theorem

$$f(x) = (x-1)(x-2)(x-3) \text{ in } [0, 4]$$

$$\text{let } f(x) = (x^2 - 2x - x + 2)(x-3)$$

$$= (x^2 - 3x + 2)(x-3)$$

$$= x^3 - 3x^2 - 3x^2 + 9x + 2x - 6$$

$$f(x) = x^3 - 6x^2 + 11x - 6$$

(i) $f(x) = x^3 - 6x^2 + 11x - 6$ is polynomial in x it is continuous for every value of x

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$\Rightarrow f(x)$ is differentiable on $(1, 2)$

hence $f(x)$ satisfies two conditions of Lagrange's theorem

$\therefore$ $\exists$ at least one point $c \in (a, b)$

$$\text{such that } f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\begin{aligned} f(b) &= f(4) = (4)^3 - 6(4)^2 + 11(4) - 6 \\ &= 64 - 96 + 44 - 6 \\ &= 108 - 102 \\ &= 6 \end{aligned}$$

$$f(a) = f(0) = 0 - 6(0) + 11(0) - 6 \\ = -6$$

$$f'(c) = 3c^2 - 12c + 11$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$3c^2 - 12c + 11 = \frac{6 - (-6)}{4 - 0}$$

$$3c^2 - 12c + 11 = \frac{12}{4} = 3$$

$$3c^2 - 12c + 8 = 0$$

$$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Put } a = 3 \quad b = -12 \quad c = 8$$

$$c = \frac{-(-12) \pm \sqrt{144 - 4 \times 3 \times 8}}{2 \times 3}$$

$$= \frac{12 \pm \sqrt{48}}{6}$$

$$= \frac{12 \pm \sqrt{16 \times 3}}{6}$$

$$\boxed{c = \frac{12 \pm 4\sqrt{3}}{6}} //$$

or

$$c = \frac{4(3 \pm \sqrt{3})}{6}$$

$$\boxed{c = \frac{6 \pm 2\sqrt{3}}{3}} //$$

Name :- Kavari Melinaiketi

UUCMS No :- UISIKM22S0001

Subject :- Mathematics

Topic :- Bisection method / Bolzano method

Sem :- IIIrd sem

Bisection Method / Bolzano Method

To find the root of an equation $f(x)=0$ is the interval $[a, b]$

Let $f(a)$ and $f(b)$ are of opposite signs. So that $f(a) \cdot f(b) < 0$.

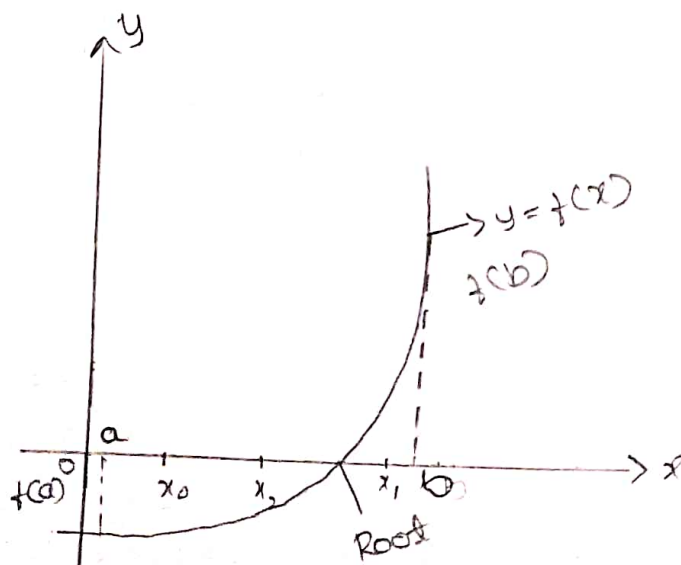
Let x_0 be the midpoint of interval (a, b) then approximate root is defined by $x_0 = \frac{a+b}{2}$

If $f(x_0) = 0$ then x_0 is the designed root of $f(x) = 0$
However if $f(x_0) \neq 0$, then root may be in between $[a, x_0]$ or $[x_0, b]$

If the root lies in the interval (a, x_0) then $f(a) \cdot f(x_0) < 0$ and x_1 is the midpoint of (a, x_0) . we define next approximation by $x_1 = \frac{a+x_0}{2}$

If the root lies in the interval (x_0, b) then $f(x_0) \cdot f(b) < 0$ and x_2 be the midpoint of (x_0, b)
i.e. $x_2 = \frac{x_0+b}{2}$

This process is continued until we get desired accuracy.



Example;

1) Find the root of equation $x^3 - 4x - 9 = 0$ using Bisection method in four iteration.

$$\text{Let } f(x) = x^3 - 4x - 9$$

$$f(0) = 0 - 0 - 9 = -9 \text{ (-ve)}$$

$$f(1) = 1^3 - 4 - 9 = -12 \text{ (-ve)}$$

$$f(2) = 2^3 - 4(2) - 9 = -9 \text{ (-ve)}$$

$$f(3) = 3^3 - 4(3) - 9 = 6 \text{ (+ve)}$$

$$\text{Since } f(2) \cdot f(3) < 0$$

$\therefore$ The root lies between 2 and 3

$$1^{\text{st}} \text{ approximation} = x_1 = \frac{2+3}{2} = 2.5$$

$$\begin{aligned} \text{Now } f(x_1) &= f(2.5) = (2.5)^3 - 4(2.5) - 9 \\ &= 15.625 - 10 - 9 \\ &= -3.375 \text{ (-ve)} \end{aligned}$$

The root lies b/w 2.5 and 3

$$2^{\text{nd}} \text{ approximation} = x_2 = \frac{2.5+3}{2} = 2.75$$

$$\begin{aligned} f(x_2) &= f(2.75) = (2.75)^3 - 4(2.75) - 9 \\ &= 0.7969 \text{ (+ve)} \end{aligned}$$

The root lies b/w 2.5 & 2.75

$$3^{\text{rd}} \text{ approximation} = x_3 = \frac{2.5+2.75}{2} = 2.625$$

$$\begin{aligned} f(x_3) &= f(2.625) = (2.625)^3 - 4(2.625) - 9 \\ &= -1.4121 \text{ (-ve)} \end{aligned}$$

The root lies between 2.625 and 2.75

$$4^{\text{th}} \text{ approximation} = x_4 = \frac{2.625+2.75}{2} = 2.6875 \text{ (+ve)}$$

Iteration Method

Let $f(x)=0$ be a given equation. It can be expressed in the form of $x=\phi(x)$.

Let $x=x_0$ be the initial approximation to the desired root.

Then the first & successive approximation to the root can be obtained as

$$x_1 = \phi(x_0), x_2 = \phi(x_1); x_3 = \phi(x_2) \dots x_n = \phi(x_{n-1})$$

And the iteration method is convergent.

$$\text{If } |\phi'(x)| < 1 \text{ i.e., } |\phi'(x)| < 1$$

Example:

- 1) Find the real root of $\cos x = 3x - 1$ correct to four decimal places using Iteration method.

$$\text{Let } f(x) = \cos x - 3x + 1$$

$$f(0) = 1 - 3(0) + 1 = 2 \text{ (+ve)}$$

$$f(1) = \cos(1) - 3(1) + 1 = -1.4597 \text{ (-ve)}$$

The root lies b/w 0 and 1

$$x_0 = \frac{0+1}{2} = 0.5 \text{ be initial approximation.}$$

Rewrite above eqⁿ in the form of $x = \phi(x)$

$$3x = \cos x + 1$$

$$x = \frac{\cos x + 1}{3} = \phi(x)$$

$$\phi'(x) = \frac{-\sin x}{3}$$

$$\therefore |\phi'(x)| < 1$$

$\therefore$ The iteration method can be applied

$$x_{n+1} = \phi(x_n), \quad n=0, 1, 2, \dots \quad x_0 = 0.5$$

$$x_1 = \phi(x_0) = \frac{\cos x_0 + 1}{3} = \frac{\cos(0.5) + 1}{3} = 0.6258$$

$$x_2 = \phi(x_1) = \frac{\cos x_1 + 1}{3} = \frac{\cos(0.6258) + 1}{3} = 0.6034$$

$$x_3 = \phi(x_2) = \frac{\cos x_2 + 1}{3} = \frac{\cos(0.6034) + 1}{3} = 0.6078$$

$$x_4 = \phi(x_3) = \frac{\cos x_3 + 1}{3} = \frac{\cos(0.6078) + 1}{3} = 0.6069$$

$$x_5 = \phi(x_4) = \frac{\cos x_4 + 1}{3} = \frac{\cos(0.6069) + 1}{3} = 0.6071$$

$$x_6 = \phi(x_5) = \frac{\cos x_5 + 1}{3} = \frac{\cos(0.6071) + 1}{3} = 0.6071$$

Name :- Krupa Biradar

UUCMS No :- U151CM2130001

Subject :- Mathematics

topic :- Angle b/w radius vector & tangent

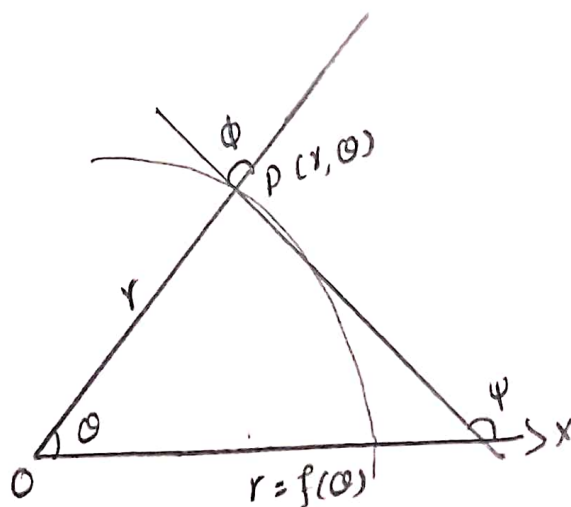
Sem :- IVth sem

Angle between the Radius vector & tangent

For the $r=f(\theta)$ at any point (r, θ) the angle ϕ between the radius vector and the tangent is given by.

$$\tan \phi = \frac{r d\theta}{dr}$$

Consider the angle ϕ between the radius vector and the tangent to a curve $r=f(\theta)$ is given in polar co-ordinates as shown in the above figure.



Let $P(r, \theta)$ be any point on the curve $r=f(\theta)$ and the tangent line to the curve $r=f(\theta)$ makes an angle ψ with respect to the x -axis and an angle ϕ with respect to the radial line at the point of tangency.

consider $\psi = \theta + \phi$ then $r = f(\theta)$ is given in polar co-ordinates by $x = r \cos \theta$ $y = r \sin \theta$ — (1)
with associated derivatives given by,

$$\frac{dx}{d\theta} = r \sin \theta + \cos \theta \frac{dr}{d\theta} \quad \frac{dy}{d\theta} = r \cos \theta + \sin \theta \frac{dr}{d\theta} \quad \text{--- (2)}$$

from the tangent of the problem in which it is evident that $2\pi - \phi - \theta = 2\pi - \psi$ we have that $\phi = \psi - \theta$ and consequently using a familiar multiple angle formula. From trigonometry that

$$\tan \phi = \tan(\psi - \theta) = \frac{\tan \psi - \tan \theta}{1 + \tan \psi \tan \theta}$$

Since the tangent line to the curve $r(\theta)$ make an angle ϕ with respect to the x-axis we have that.

$$\tan \psi = \frac{dy/d\theta}{dx/d\theta} \quad \text{and trivially from the}$$

$$\text{geometry that} \quad \tan \theta = \frac{y}{x}$$

- Substitute: these into (3) gives

$$\tan \phi = \frac{\frac{dy/d\theta}{dx/d\theta}}{1 + \left[\frac{y}{x}\right] \frac{dy/d\theta}{dx/d\theta}} = \frac{x \frac{dy}{d\theta} - y \frac{dx}{d\theta}}{x \frac{dx}{d\theta} + y \frac{dy}{d\theta}}$$

$$= \frac{x \left[r \cos \theta + r \sin \theta \frac{dr}{d\theta} \right] - y \left[-r \sin \theta + \cos \theta \frac{dr}{d\theta} \right]}{x \left[-r \sin \theta + \cos \theta \frac{dr}{d\theta} \right] + y \left[r \cos \theta + \sin \theta \frac{dr}{d\theta} \right]} \quad \text{--- (4)}$$

Substituting equation (4) in (4) we get

$$\tan \phi = \frac{r \cos \theta \left\{ r \cos \theta + \sin \theta \left(\frac{dr}{d\theta} \right) \right\} - r \sin \theta \left\{ -r \sin \theta + \cos \theta \left(\frac{dr}{d\theta} \right) \right\}}{r \cos \theta \left\{ -r \sin \theta + \cos \theta \left(\frac{dr}{d\theta} \right) \right\} + r \sin \theta \left\{ r \cos \theta + \sin \theta \left(\frac{dr}{d\theta} \right) \right\}}$$

$$\tan \phi = \frac{r^2}{r \frac{dr}{d\theta}}$$

$$\tan \phi = \frac{r}{\frac{dr}{d\theta}} \quad \text{or} \quad \tan \phi = r \frac{d\theta}{dr}$$

Find the angle b/w radius vector and tangent to the curve $r = a(1 + \cos \theta)$ at $\theta = \pi/6$

⇒ Given

$$r = a(1 + \cos \theta)$$

Applying log on b.s

$$\log r = \log a + \log (1 + \cos \theta)$$

diff. w. r. t. θ

$$\frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{1 + \cos \theta} \times (-\sin \theta)$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{-\sin \theta}{1 + \cos \theta}$$

$$\frac{r d\theta}{dr} = -\frac{1 + \cos \theta}{\sin \theta}$$

$$\tan \phi = -\frac{(1 + \cos(\pi/6))}{\sin \theta}$$

$$\tan \phi = -\frac{(1 + \cos \pi/6)}{\sin \pi/6}$$

$$\tan \phi = -\frac{(1 + \sqrt{3}/2)}{1/2}$$

$$\tan \phi = -\frac{(2 + \sqrt{3})}{\frac{1}{2}}$$

$$\tan \phi = [-(2 + \sqrt{3})]$$

find the angle between radius vector and tangent to the curve $r = a e^{\theta \tan \alpha}$

$\Rightarrow$ Given $r = a e^{\theta \tan \alpha}$

Applying $\log$ on both side

$$\log r = \log a + \log e^{\theta \cot \alpha}$$

$$\log r = \log a + \theta \cot \alpha \log e$$

diff. w. r. t. θ

$$\frac{1}{r} \frac{dr}{d\theta} = 0 + \cot \alpha (1)$$

$$\frac{r d\theta}{dr} = \frac{1}{\cot \alpha}$$

$$\tan \phi = \tan \alpha$$

$$\boxed{\phi = \alpha}$$

find the angle b/w radius vector & tangent to the curve $\frac{2a}{r} = 1 - \cos \theta$

$$\Rightarrow \text{Given} = \frac{2a}{r} = 1 - \cos \theta$$

Applying $\log$ on both side

$$\log \frac{2a}{r} = \log (1 - \cos \theta)$$

$$\log 2a - \log r = \log (1 - \cos \theta)$$

$$\log r = \log 2a - \log (1 - \cos \theta)$$

diff. w. r. to θ .

$$\frac{1}{r} \frac{dr}{d\theta} = 0 - \frac{1}{(1 - \cos \theta)} (+\sin \theta)$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{- (1 - \cos \theta)}{\sin \theta}$$

$$\frac{r d\theta}{dr} = \frac{- \sin \theta}{1 - \cos \theta}$$

$$\frac{r d\theta}{dr} = \frac{- 2 \sin^2 \theta / 2}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}$$

$$\frac{r d\theta}{dr} = - \frac{\tan \theta}{2}$$

$$\tan \phi = - \tan \theta$$

$$\tan \phi = \tan (180 - \frac{\theta}{2})$$

$$\tan \phi = \tan (\pi - \theta/2)$$

$$\phi = \pi - \theta/2$$

Find the angle b/w radius vector & tangent to the curve $r^n = a^m \cos m\theta$

Given: $r^m = a^m \cos m\theta$

Applying log on b.s

$$\log r^m = \log a^m + \log \cos m\theta$$

$$m \log r = m \log a + \log \cos m\theta$$

diff. w. r. t. θ

$$m \times \frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{\cos m\theta} (-\sin m\theta) \times m$$

$$\ln \frac{1}{r} \frac{dr}{d\theta} = \ln \left[\frac{-\sin m\theta}{\cos m\theta} \right]$$

$$\frac{r d\theta}{dr} = - \frac{\cos m\theta}{\sin m\theta}$$

$$\tan \phi = - \cot m\theta$$

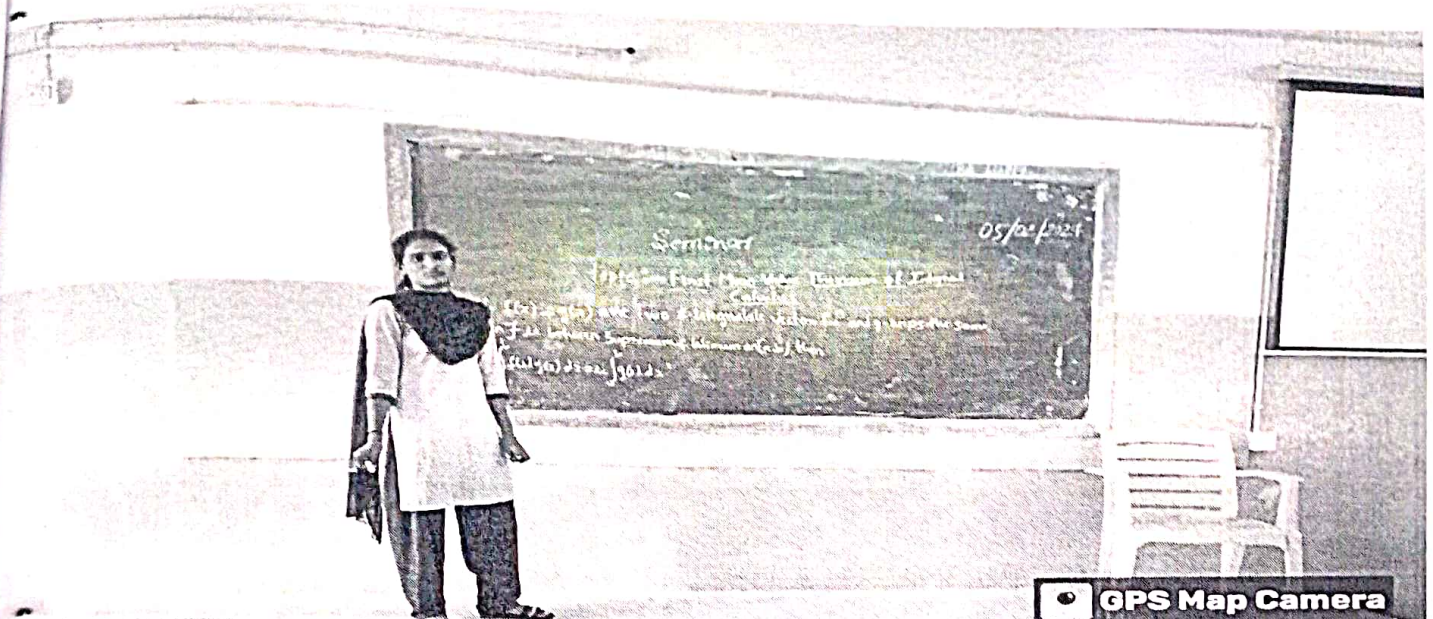
$$\tan \phi = - \tan (\pi/2 - m\theta)$$

$$\tan \phi = \tan (\pi - \{ \pi/2 - m\theta \})$$

$$\phi = \pi - (\pi/2 - m\theta)$$

$$\phi = \pi - \frac{\pi}{2} + m\theta$$

$$\phi = \frac{\pi}{2} + m\theta.$$

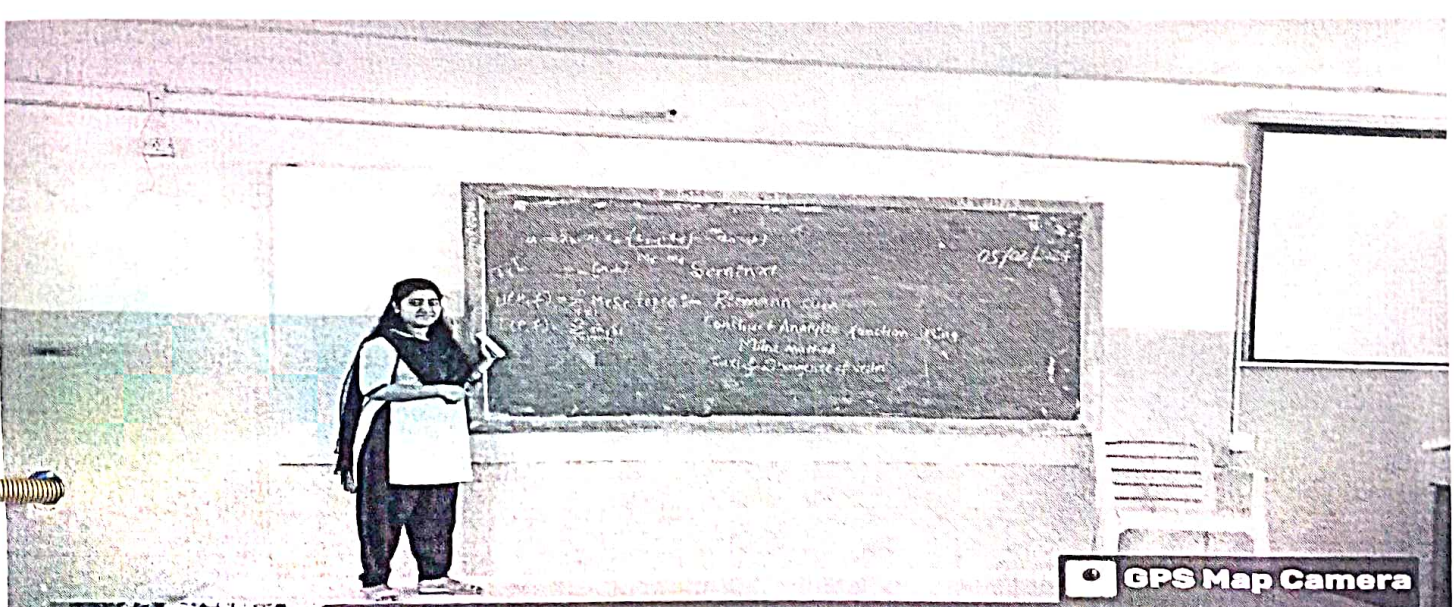


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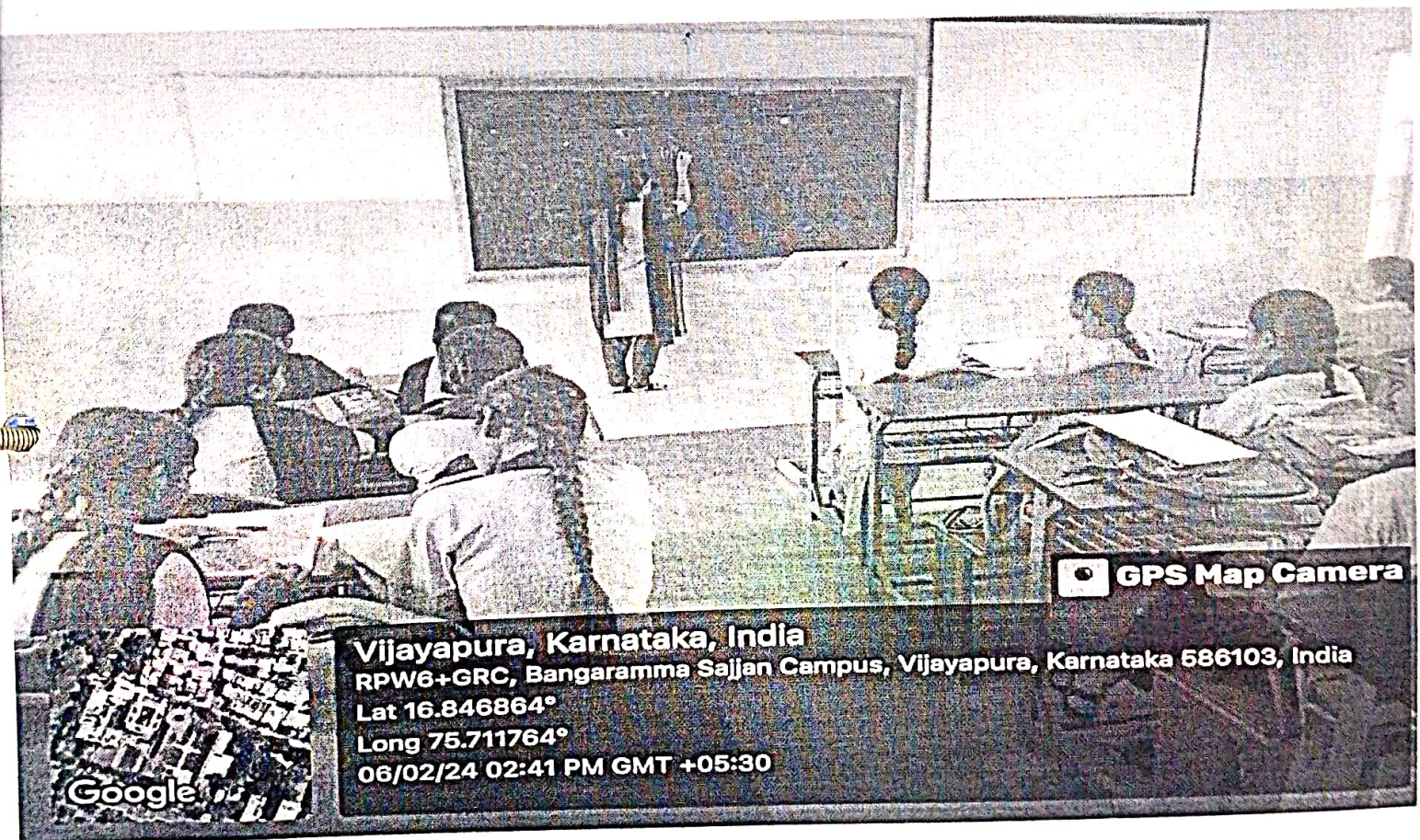


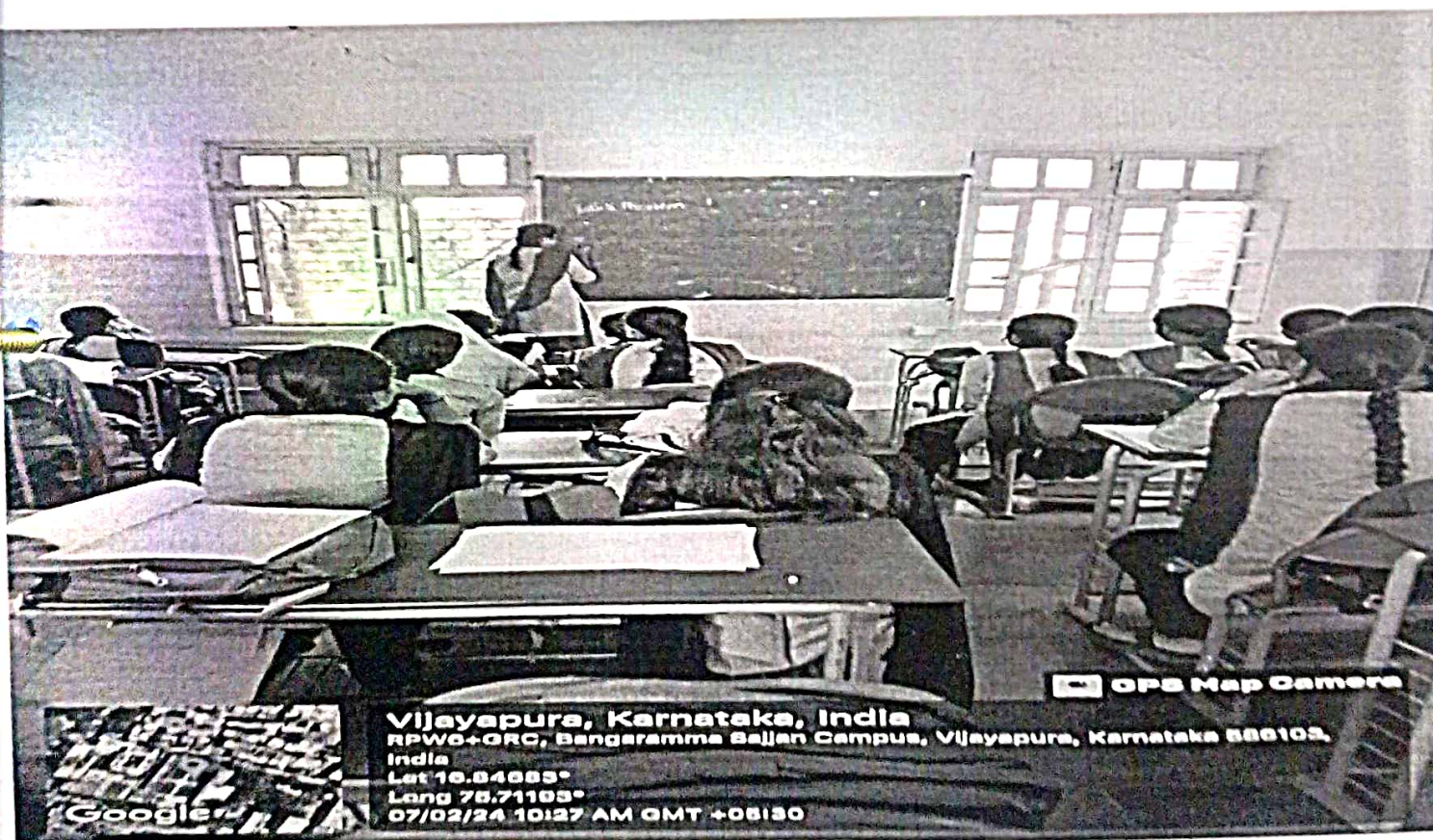
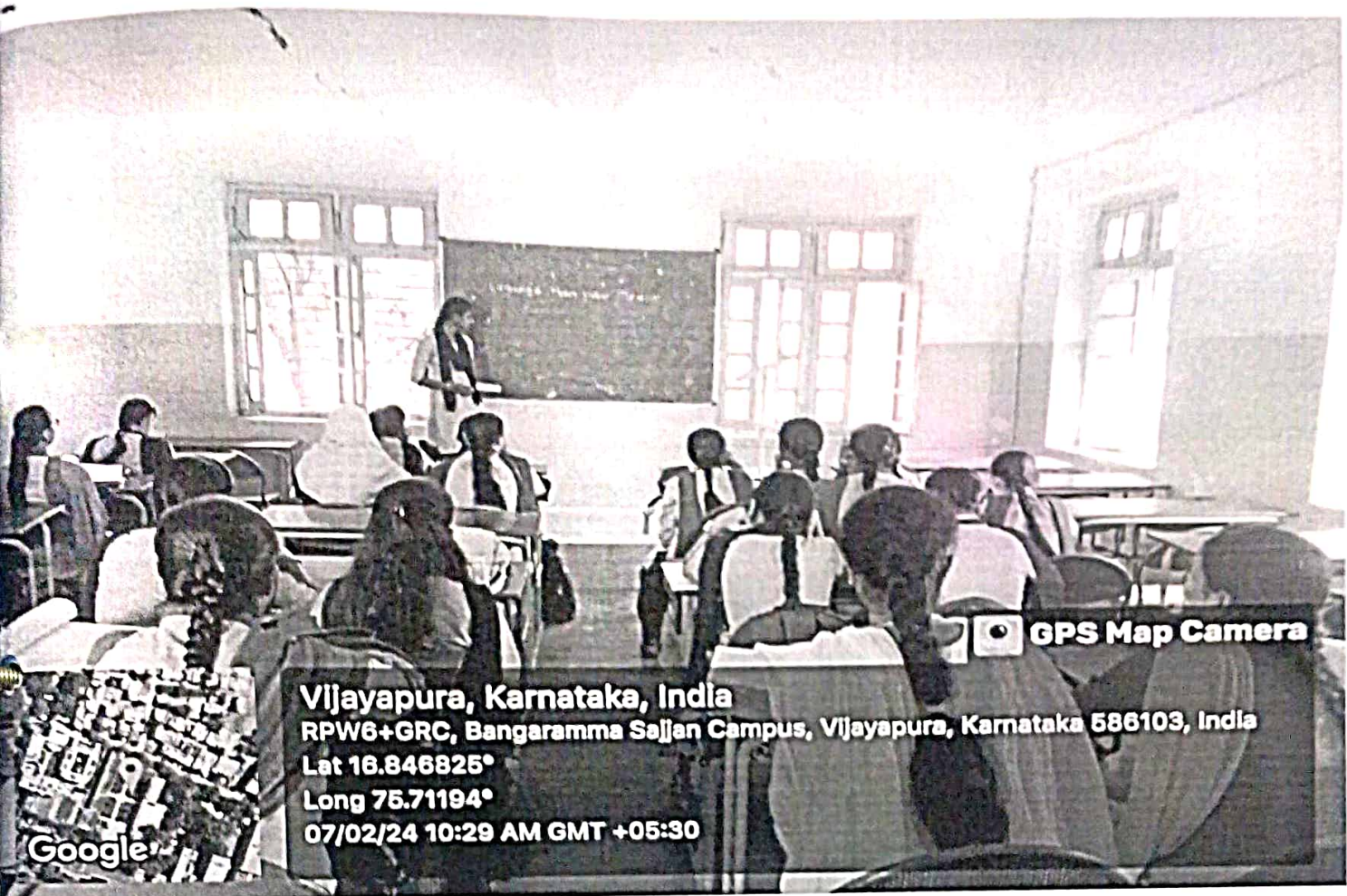
 **GPS Map Camera**

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B. L. D. E. ASSOCIATION'S
S. B. ARTS AND K. C. P. SCIENCE COLLEGE
VIJAYAPUR
DEPARTMENT OF MATHEMATICS



Date: 09/02/2024

Seminar Attendance sheet

SI. NO	UUCMS. NO	NAME OF THE STUDENTS	B.SC	Signature
1	U15KM23S0191	Ambika Sankh	I	A. J. S.
2	U15KM23S0067	Aishwarya	I	A. B. M.
3	U15KM23S0071	Danamma Bidari	I	D. M. Bidari
4	U15KM23S0132	Anita Ankalagi	I	A. G. Ankalagi
5	U15KM23S0157	Priya Madar	I	Priya.
6	U15KM23S0209	Rukmini Darveshi	I	R.R.
7	U15KM23S0213	Bhagyashree	I	Bhagya.
8	U15KM23S0243	Bhagyashree	I	B. S. C.
9	U15KM23S0302	Kaveri Hullur	I	K. H. S.
11	U15KM23S0367	Priyanka	I	P. S. H.
12	U15KM23S0366	Sneha Katti	I	S. S. Katti
13	U15KM22S0001	Kaveri Melinakeri	III	K. M.
14	U15KM22S0005	Sakshi Ajur	III	Ajunt
15	U15KM22S0009	Suma Hitnalli	III	S. H.
16	U15KM22S0022	Bhagyashree	III	B. S.
17	U15KM22S0026	Sushmita	III	S. G. Jangamshetti
18	U15KM22S0060	Mahadevi B Patil	III	M. B. Patil
19	U15KM22S0098	Sangeeta Kirasur	III	S. K.
20	U15KM22S0102	Swati Chavhan	III	S. Chavhan
21	U15KM22S0128	Bhagyashree	III	B. S.
22	U15KM22S0130	Abhishek Rathod	III	A. R.
23	U15KM22S0137	Ningaraddy	III	N. R.
24	U15KM22S0153	Jayashree	III	J. S.
25	U15KM22S0162	Akshata Badagi	III	A. R. Badagi
26	U15KM22S0165	Arpita Sajjan	III	A. S.
27	U15KM22S0167	Shivaleela	III	S. R. N.
28	U15KM22S0179	Asharani Bhali	III	A. B.
29	U15KM22S0180	Bhoomika	III	B. M.

30	U15KM22S0221	Abdul Yadagiri	III	ATHY...
31	U15KM22S0224	Sahana Sajjan	III	Sajjan.
32	U15KM22S0294	Danamma Poojari	III	Edaj
33	U15KM22S0305	Pooja Ashok Katti	III	Prati
34	U15KM22S0311	Amruta Shiroor	III	AR.
35	U15KM22S0312	Revati Nesur	III	R.R.Nesur
36	U15JU22S0058	Prema Bagalkot	III	Prema.
37	U15KM21S0001	Krupa Biradar	V	Rupa
38	U15KM21S0026	Sabale Diksheetsa	V	Sabale
39	U15KM21S0043	Nikita Biradar	V	Nikhit.
40	U15KM21S0097	Bhuvaneshwari	V	Bhuv.
41	U15KM21S0163	Prajwal Aihole	V	Prajwal
42	U15KM21S0289	Annaray Bhavikatti	V	Annaray.sg
43	U15KM21S0315	Prema S	V	Prema
44	U15KM21S0339	Bhagyanidhi	V	Bhagyanidhi
45	U15KM21S0366	Savita Bilijadar	V	Savita
46	U15KM21S0370	Kashibai Pujari	V	K.P.
47	U15KM21S0372	Spoorti Siddapur	V	Spoorti
48	U15KM21S0384	Shruti Pujari	V	S.B. Pujari
49	U15KM21S0389	Nagaveni Harawal	V	Nagaveni
50	U15KM21S0390	Aishwarya	V	Aishwarya
51	U15KM21S0396	Vidya Biradar	V	Vidya
52	U15KM21S0412	Vishal Sarur	V	Vishal
53	U15KM21S0471	Madhubaratagi	V	M.M.R.
54	U15KM21S0506	Ragini Dwivedi	V	Ragini
55	U15KM21S0562	Tejashwini	V	Tejashwini
56	U15MY21S0086	Pooja	V	Pooja


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Principal,
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Name :- Kumar Biradar

Subject :- Mathematics

UUCMS NO :- U151CM2350017

Topic :- Integrable function

Sem :- IInd sem

Theorem:-

Statement: If f and g are integrable function on $[a, b]$, then we have to prove $fg \in R[a, b]$.

Proof:

Let f and g are two integrable function on $[a, b]$ there exist a positive real number 'M' such that, $|f(x)| \leq M$,

$$|g(x)| \leq M \quad \forall x \in [a, b]$$

$$\begin{aligned} |fg(x)| &= |f(x)g(x)| \\ &= |f(x)| |g(x)| \\ &\leq M M \\ &\leq M^2 \end{aligned}$$

This shows that fg is bounded.

Let $f \in R[a, b]$ and $g \in R[a, b] \exists$ a partitions P_1 and P_2 of $[a, b]$.

$$\left. \begin{aligned} U(P_1, f) - L(P_1, f) &< \frac{\epsilon}{2M} \\ U(P_2, g) - L(P_2, g) &< \frac{\epsilon}{2M} \end{aligned} \right\} \textcircled{1}$$

$$P = P_1 \cup P_2$$

$$\left. \begin{aligned} U(P, f) - L(P, f) &< \frac{\epsilon}{2M} \\ U(P, g) - L(P, g) &< \frac{\epsilon}{2M} \end{aligned} \right\} \textcircled{2}$$

Let M_r and m_r are the function of fg .

Let M_r and m_r are the supremum and Infimum of function fg on I_r .

M_r' and m_r' are the supremum and Infimum of function ' f ' on I_r .

M_r'' and m_r'' are the supremum and Infimum of function ' g ' on I_r .

Consider,

$$\begin{aligned} |fg(x) - fg(y)| &= |f(x)g(x) - f(y)g(y)| \\ &= |f(x)g(x) - f(y)g(x) + f(y)g(x) - f(y)g(y)| \\ &= |g(x)(f(x) - f(y)) + f(y)(g(x) - g(y))| \end{aligned}$$

$$\begin{aligned} |fg(x) - fg(y)| &\leq |g(x)(f(x) - f(y))| + |f(y)(g(x) - g(y))| \\ &\leq |g(x)| |f(x) - f(y)| + |f(y)| |g(x) - g(y)| \\ &\leq M |f(x) - f(y)| + M |g(x) - g(y)| \end{aligned}$$

$$M_r - m_r \leq M(M_r' - m_r') + M(M_r'' - m_r'')$$

Multiply δ_r and Σ on both side.

$$\sum_{r=1}^n (M_r - m_r) \delta_r \leq M \sum_{r=1}^n (M_r' - m_r') \delta_r + M \sum_{r=1}^n (M_r'' - m_r'') \delta_r$$

$$\begin{aligned} \sum_{r=1}^n M_r \delta_r - \sum_{r=1}^n m_r \delta_r &\leq M \left(\sum_{r=1}^n M_r' \delta_r - \sum_{r=1}^n m_r' \delta_r \right) \\ &\quad + M \left(\sum_{r=1}^n M_r'' \delta_r - \sum_{r=1}^n m_r'' \delta_r \right) \end{aligned}$$

$$\begin{aligned} U(P, fg) - L(P, fg) &\leq M(U(P, f) - L(P, f)) \\ &\quad + M(U(P, g) - L(P, g)) \end{aligned}$$

from ②

$$U(P, fg) - L(P, fg) \leq M \frac{\epsilon}{2M} + M \frac{\epsilon}{2M} \Rightarrow \frac{\epsilon}{2}$$

$$U(P, fg) - L(P, fg) < \epsilon$$

Hence $fg \in R[a, b]$.

Theorem:-

Statement: If f and g are two functions both bounded and integrable on $[a, b]$ and there exist a number ' t ' > 0 , such that $|g(x)| > t$, $\forall x \in [a, b]$ then we have to prove f/g is bounded and integrable on $[a, b]$.

Proof:-

Let f and g are bounded function on $[a, b]$. $\exists$ a positive real number k , such that,

$$|f(x)| \leq k, |g(x)| \leq k, |g(x)| \geq t, \frac{1}{|g(x)|} \leq \frac{1}{t}$$

$$\left| \frac{f(x)}{g(x)} \right| = \left| \frac{f(x)}{g(x)} \right| \leq \frac{k}{t}$$

This shown that f/g is bounded.

Let $f \in R[a, b]$ and $g \in R[a, b]$. $\exists$ a partition P_1 and P_2 of $[a, b]$.

$$\left. \begin{aligned} U(P_1, f) - L(P_1, f) &< t^2 \frac{\epsilon}{2k} \\ U(P_2, g) - L(P_2, g) &< t^2 \frac{\epsilon}{2k} \end{aligned} \right\} \textcircled{1}$$

$$\begin{aligned} P &= P_1 \cup P_2 \\ \left. \begin{aligned} U(P, f) - L(P, f) &< t^2 \frac{\epsilon}{2k} \\ U(P, g) - L(P, g) &< t^2 \frac{\epsilon}{2k} \end{aligned} \right\} \textcircled{2} \end{aligned}$$

Let M_r and m_r are the supremum and infimum of f/g on I_r .

Let M_r' and m_r' are the supremum and infimum of f on I_r .

Let M_r'' and m_r'' are the supremum and infimum of g on I_r .

Consider, $\alpha, \beta \in I_r$.

$$\begin{aligned} \left| \frac{f(\beta)}{g(\beta)} - \frac{f(\alpha)}{g(\alpha)} \right| &= \left| \frac{f(\beta)g(\alpha) - f(\alpha)g(\beta)}{g(\alpha)g(\beta)} \right| \\ &= \frac{f(\beta)g(\alpha) - f(\alpha)g(\beta) - f(\alpha)g(\alpha) + f(\alpha)g(\alpha)}{g(\alpha)g(\beta)} \\ &= \frac{g(\alpha)(f(\beta) - f(\alpha)) - f(\alpha)(g(\beta) - g(\alpha))}{g(\alpha) \cdot g(\beta)} \\ &= \left| \frac{g(\alpha)(f(\beta) - f(\alpha))}{g(\alpha)g(\beta)} \right| + \left| \frac{f(\alpha)(g(\beta) - g(\alpha))}{g(\alpha)g(\beta)} \right| \\ &= \frac{|g(\alpha)| |f(\beta) - f(\alpha)|}{|g(\alpha)| |g(\beta)|} + \frac{|f(\alpha)| |g(\beta) - g(\alpha)|}{|g(\alpha)| |g(\beta)|} \end{aligned}$$

$$\left| \frac{f(\beta)}{g(\beta)} - \frac{f(\alpha)}{g(\alpha)} \right| \leq \frac{k |f(\beta) - f(\alpha)|}{t^2} + \frac{k |g(\beta) - g(\alpha)|}{t^2}$$

$$M_r - m_r \leq \frac{k}{t^2} (M_r' - m_r') + \frac{k}{t^2} (M_r'' - m_r'')$$

Multiply δr and taking Σ on both side.

$$\sum_{r=1}^n (M_r - m_r) \delta r \leq \frac{k}{t^2} \sum_{r=1}^n (M_r' - m_r') \delta r + \frac{k}{t^2} \sum_{r=1}^n (M_r'' - m_r'') \delta r$$

$$\sum_{r=1}^n M_r \delta r - \sum_{r=1}^n m_r \delta r \leq \frac{k}{t^2} \left(\sum_{r=1}^n M_r' \delta r - \sum_{r=1}^n m_r' \delta r \right) + \frac{k}{t^2} \left(\sum_{r=1}^n M_r'' \delta r - \sum_{r=1}^n m_r'' \delta r \right)$$

$$U(P, f/g) - L(P, f/g) \leq \frac{K}{t^2} (U(P, f) - L(P, f)) \\ + \frac{K}{t^2} (U(P, g) - L(P, g))$$

From eqⁿ ②

$$U(P, f/g) - L(P, f/g) \leq \frac{K}{t^2} \left(t^2 \frac{\epsilon}{2K} \right) + \frac{K}{t^2} \left(t^2 \frac{\epsilon}{2K} \right)$$

$$U(P, f/g) - L(P, f/g) \leq \frac{\epsilon}{2} + \frac{\epsilon}{2}$$

$$U(P, f/g) - L(P, f/g) \leq \frac{2\epsilon}{2}$$

$$U(P, f/g) - L(P, f/g) \leq \epsilon$$

Hence $f/g \in R[a, b]$,

Name :- Sahana Mallappa

UUCMS NO :- ULS14M2280125

Subject :- Mathematics

Topic :- Echelon form of matrix

Sem :- IV sem

Echelon Form of Matrix :-

A matrix A is said to be in Echelon form if

- i) all the non-zero rows if any precede the zero rows

- ii) The Number of zero's preceding the first non-zero element in a row is less than the Succeeding rows

- iii) The first non-zero element in a row is unity

or

A non zero matrix A is said to be in a row Echelon form if the following conditions Satisfies

- i) all the zero rows are below non-zero rows

- ii) the first non zero entry in any non zero row is one.

- iii) Any $M \times N$ (where $M \geq 2$ and $N \geq 2$) Matrix is said to be in Row Echelon form.

Example

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Column Echelon Form of Matrix

- i) Any $M \times N$ (where $M \geq 2$ and $N \geq 2$) Matrix is said to be in Column Echelon Form if all the following 3 conditions are true
 - a) All the zero values must precede any non-zero values for all the columns
 - b) All the non-zero values must precede any zero values for all the rows/co-vectors
 - c) First row/co-vector must have one and only one non-zero value. Any succeeding rows have at least as many non-zero values as the previous row/co-vector and at most one non-zero value more than the previous row/co-vectors

Hence, the Column Echelon Matrix have all the zero values concentrated at the top and right corner of the matrix while the non-zero values are concentrated at the bottom and left corner of the matrix.

Examples of Reduction of Echelon Form of Matrix

① Reduce the following matrix to its Echelon Form. $A =$

$$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

Apply $R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 1 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

$$R_2 = R_2 - 2R_1 \quad R_3 = R_3 + R_1$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & -5 & 1 \\ 0 & 4 & 4 \end{bmatrix}$$

$$R_3 = \frac{1}{4}R_3$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & -5 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_3 = 5R_3 + R_2$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 0 & -5 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

$$R_1 = R_1/2 \quad R_2 = R_2/-5 \quad R_3 = R_3/6$$

$$\begin{bmatrix} 1 & 3/2 & -1/2 \\ 0 & 1 & -1/5 \\ 0 & 0 & 1 \end{bmatrix}$$

clearly this is the Echelon form of the matrix
 A Number of non zero is 3.

② Reduce the following matrix to Echelon Form $A =$

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$$

$$R_3 = R_3 - 2R_1 \quad R_4 = R_4 - 4R_1$$

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & -5 & 5 & -3 \\ 0 & -11 & 5 & -3 \end{bmatrix}$$

$$R_4 = R_4 - R_3 \quad R_3 = R_3 + R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2 = R_2 / 11$$

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -5/11 & 3/11 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Clearly this is the Echelon form of the matrix A.

Number of non zero row is 2

③ Reduce the following matrix to its Echelon form and also A =

$$\begin{bmatrix} 8 & 2 & 1 & 5 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 1 & 3 \\ 5 & 1 & 1 & 4 \end{bmatrix}$$

Given A =

$$\begin{bmatrix} 8 & 2 & 1 & 5 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 1 & 3 \\ 5 & 1 & 1 & 4 \end{bmatrix}$$

$$C_1 \leftrightarrow C_3$$

$$\begin{bmatrix} 1 & 2 & 8 & 6 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 3 & 3 \\ 1 & 1 & 5 & 4 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_1 \quad R_4 \rightarrow R_4 - R_1$$

$$\begin{bmatrix} 1 & 2 & 8 & 6 \\ 0 & 1 & 2 & 1 \\ 0 & -2 & -5 & -3 \\ 0 & -1 & -3 & -2 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + 2R_2 \quad R_4 \rightarrow R_4 + R_2$$

$$\begin{bmatrix} 1 & 2 & 8 & 6 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3$$

$$\begin{bmatrix} 1 & 2 & 8 & 6 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 \rightarrow \frac{R_3}{-1}$$

$$\begin{bmatrix} 1 & 2 & 8 & 6 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Clearly this is the Echelon form of the matrix A
 $\therefore$ Number of non zero rows are 3

BLDEA'S

S B Arts and

KCP Science

College Vijayapur

Seminar on : Numerical Analysis

Name : Bhagyanidhi Budihal

Sub : Mathematics (II) (B div)

Class : BSc VI sem

VUCMS NO: U15KM2150339

Seen

1) Find a real root of Eqn $x^3 - 2x - 5 = 0$ by the method of false position correct to 3 decimal places

let $f(x) = x^3 - 2x - 5$ then

$$f(0) = -5, \quad f(1) = -6, \quad f(2) = -1, \quad f(3) = 16$$

$\therefore$ the root lies in $(2, 3)$ & it may be observed that value of $f(x)$ at $x=2$ being -1 is nearer to zero compared to $f(3) = 16$, so that we expect root lies in right neighbourhood of 2

$$\text{Now } f(2.1) = 0.061 > 0$$

$\therefore$ the roots lie b/w 2 & 2.1

I appro-

$$\text{here } a = 2 \quad b = 2.1$$

$$f(a) = -1 \quad f(b) = 0.061$$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = \frac{2(0.061) - 2.1(-1)}{0.061 + 1} = 2.0942$$

Now,

$$\begin{aligned} f(2.0942) &= 9.18447 - 4.1884 - 5 \\ &= -0.0039 < 0 \end{aligned}$$

$\therefore$ the root lies b/w 2.0942 & 2.1

II appro-

$$x_2 = \frac{2.0942(0.061) - 2.1(-0.0039)}{0.061 + 0.0039} = 2.094$$

Comparing x_1 & x_2 we observe that the values are same upto 3 decimal places

hence approximate root is 2.094

2) Solve by Gauss-Jordan Method

$$2x + 5y + 7z = 52$$

$$2x + y - z = 0$$

$$x + y + z = 9$$

The given system of eqn can be written in the form of $AX = B$

$$\begin{bmatrix} 2 & 5 & 7 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 52 \\ 0 \\ 9 \end{bmatrix}$$

Consider the augmented matrix $[A:B]$

$$[A:B] = \left[\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \\ 1 & 1 & 1 & 9 \end{array} \right]$$

$$R_1 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \\ 2 & 5 & 7 & 52 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1 \quad \& \quad R_3 \rightarrow R_3 - 2R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 3 & 5 & 34 \end{array} \right]$$

$$R_1 \rightarrow R_1 + R_2 \quad \& \quad R_3 \rightarrow R_3 + 3R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array} \right]$$

$$R_3 \rightarrow (-1/4) R_3$$

$$[A:B] \sim \begin{bmatrix} 1 & 0 & 2 & : & -9 \\ 0 & -1 & -3 & : & -18 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

$$R_1 \rightarrow R_1 + 2R_3$$

$$R_2 \rightarrow R_2 + 3R_3$$

$$[A:B] \sim \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & -1 & 0 & : & -3 \\ 0 & 0 & 1 & : & 5 \end{bmatrix}$$

$$x=1, y=3, z=5$$

Hence the required solution is

$$x=2, y=3, z=5.$$

Find the Polynomial by using Newton forward interpolation formula.

$x:$	0	1	2	3
$f(x):$	1	2	1	10

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
1	2	1		
2	1	-1	-2	
3	10	9	10	12

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write Newton forward interpolation formula

$$f(x) = f(a) + u \cdot \frac{\Delta f(a)}{1!} + u(u-1) \frac{\Delta^2 f(a)}{2!} + \dots + u(u-1)(u-2)\dots(u-n+1) \frac{\Delta^n f(a)}{n!}$$

here, $h=1$

$$x = a + hu$$

$$u = \frac{x-a}{h} \Rightarrow \frac{x-0}{1} = x \quad [a=0]$$

$$\boxed{u=x}$$

$$f(x) = f(0) + \frac{x \Delta f(0)}{1} + \frac{x(x-1) \Delta^2 f(0)}{2} + \frac{x(x-1)(x-2) \Delta^3 f(0)}{6}$$

$$= 1 + x(1) + \frac{x(x-1)(-2)}{2} + \frac{x(x-1)(x-2)(\frac{2}{1})}{6}$$

$$= 1 + x - (x^2 - x) + x(x^2 - 2x - x + 2)2$$

$$= 1 + x - x^2 + x + 2x(x^2 - 3x + 2)$$

$$= 1 + x - x^2 + x + 2x^3 - 6x^2 + 4x$$

$$f(x) = 2x^3 - 5x^2 + 6x + 1$$

which is required polynomial.

47 find $f'(1.5)$ from given table

x :	0.0	0.5	1.0	1.5	2.0
y :	0.3989	0.3521	0.2420	0.1295	0.0540

consider diff table

x	y	$\nabla f(x_n)$	$\nabla^2 f(x_n)$	$\nabla^3 f(x_n)$	$\nabla^4 f(x_n)$
0.0	0.3989				
		-0.0468			
0.5	0.3521		-0.0633		
		-0.1101		0.0609	
1.0	0.2420		-0.0024		-0.0215
		-0.1125		0.0394	
1.5	0.1295		0.037		
		-0.0755			
2.0	0.0540				

here $h=0.5$ $x=1.5$ $x_n=1.5$ $P = \frac{1.5-1.5}{0.5} = 0$

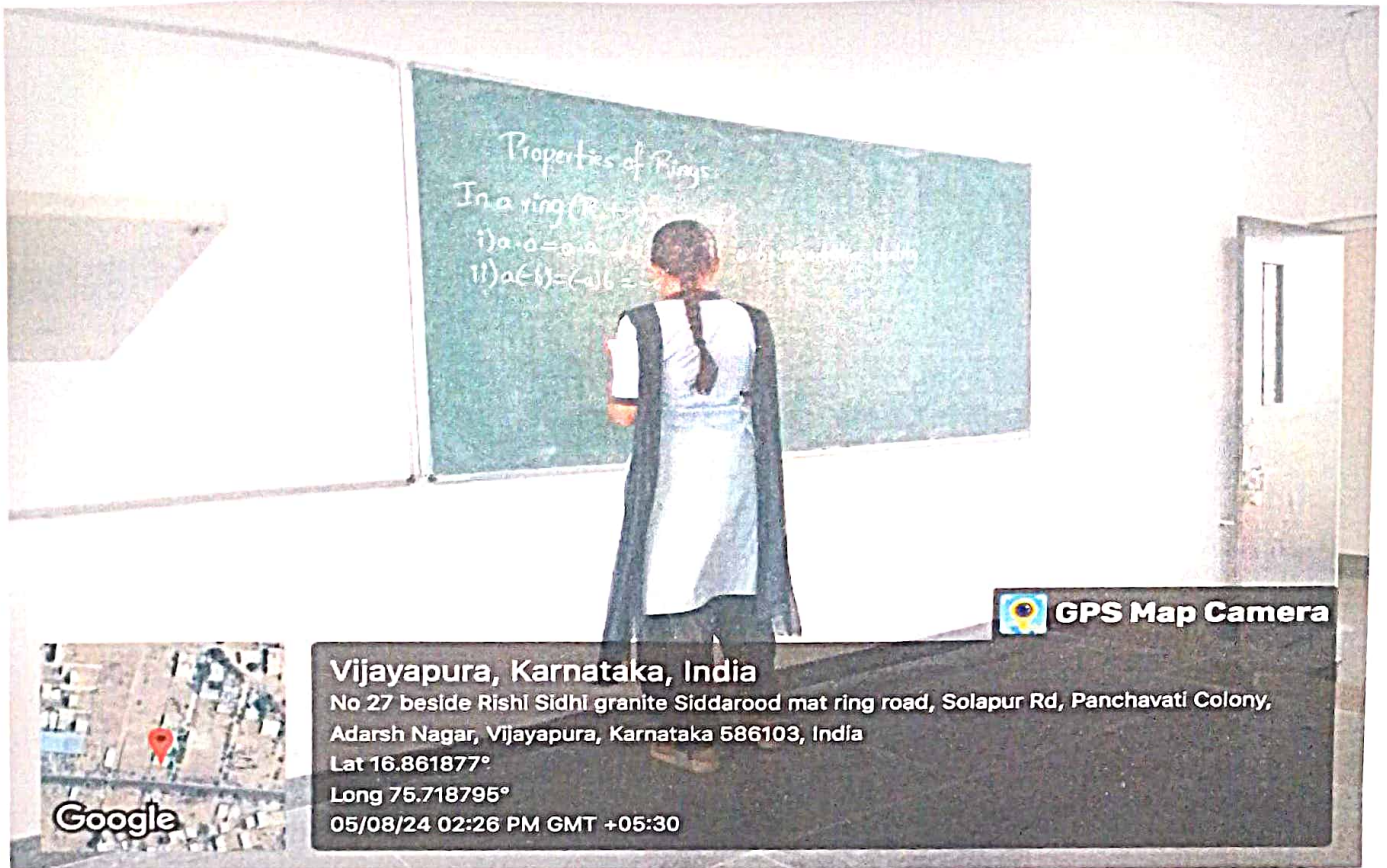
$$f'(x_n) = \frac{1}{h} \left[\nabla f(x_n) + \frac{1}{2} \nabla^2 f(x_n) + \frac{1}{3} \nabla^3 f(x_n) \right]$$

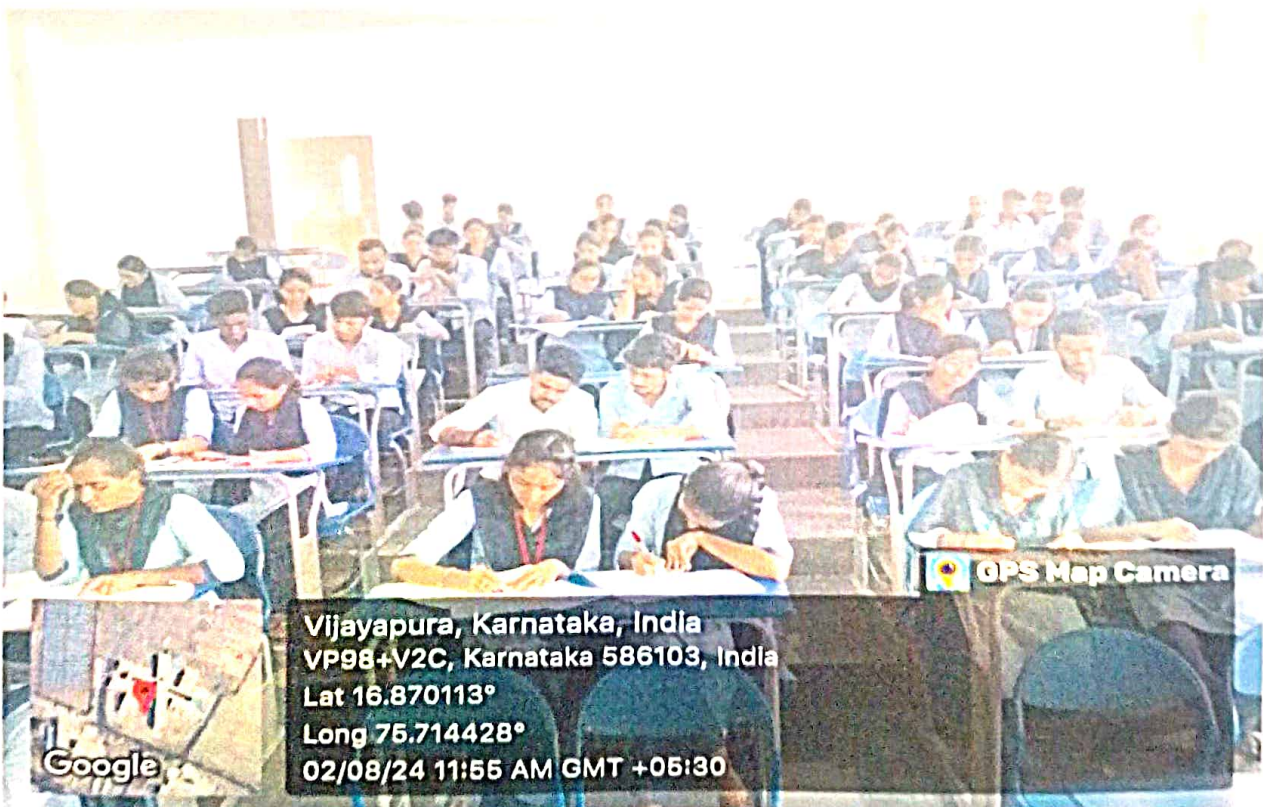
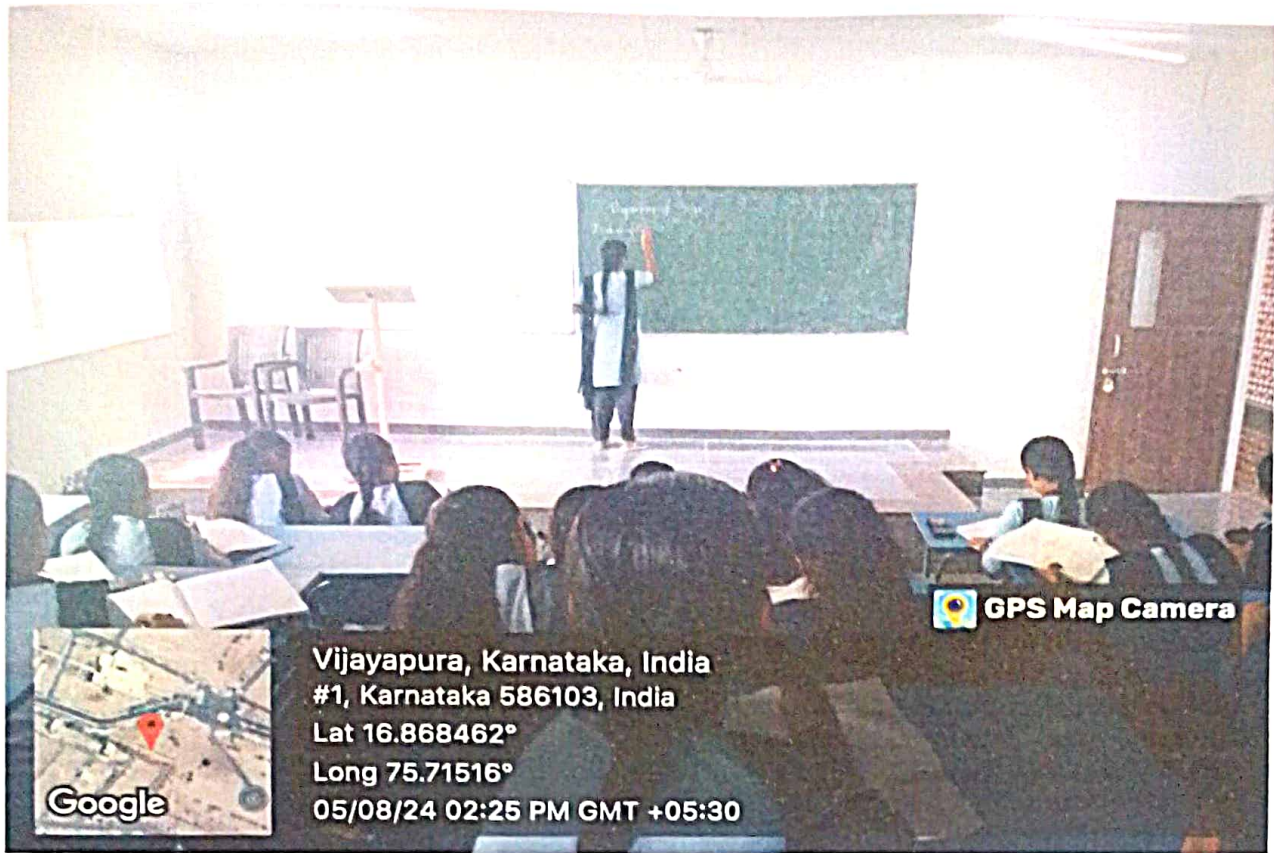
$$f'(1.5) = \frac{1}{0.5} \left[-0.1125 + \frac{1}{2} (-0.0024) + \frac{1}{3} (0.0609) \right]$$

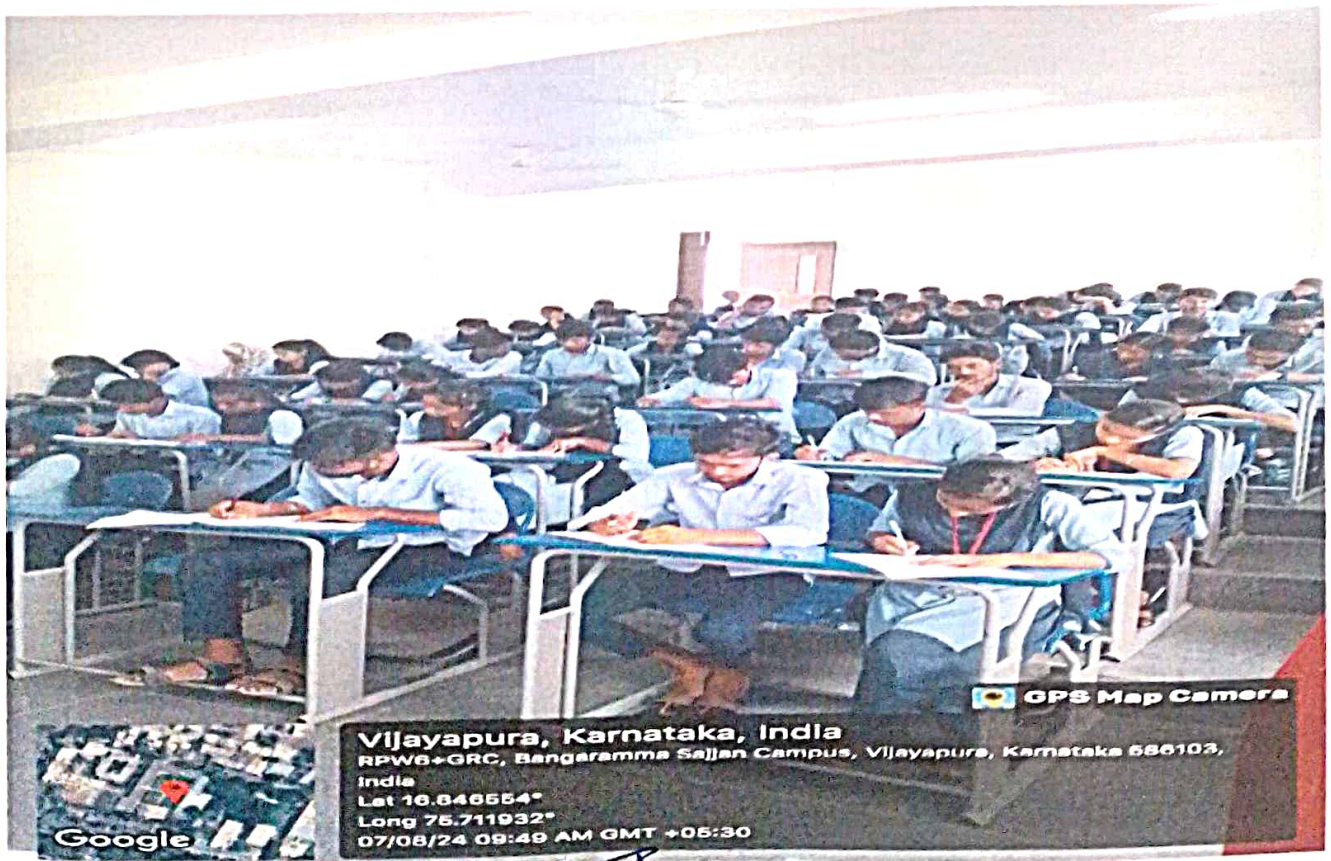
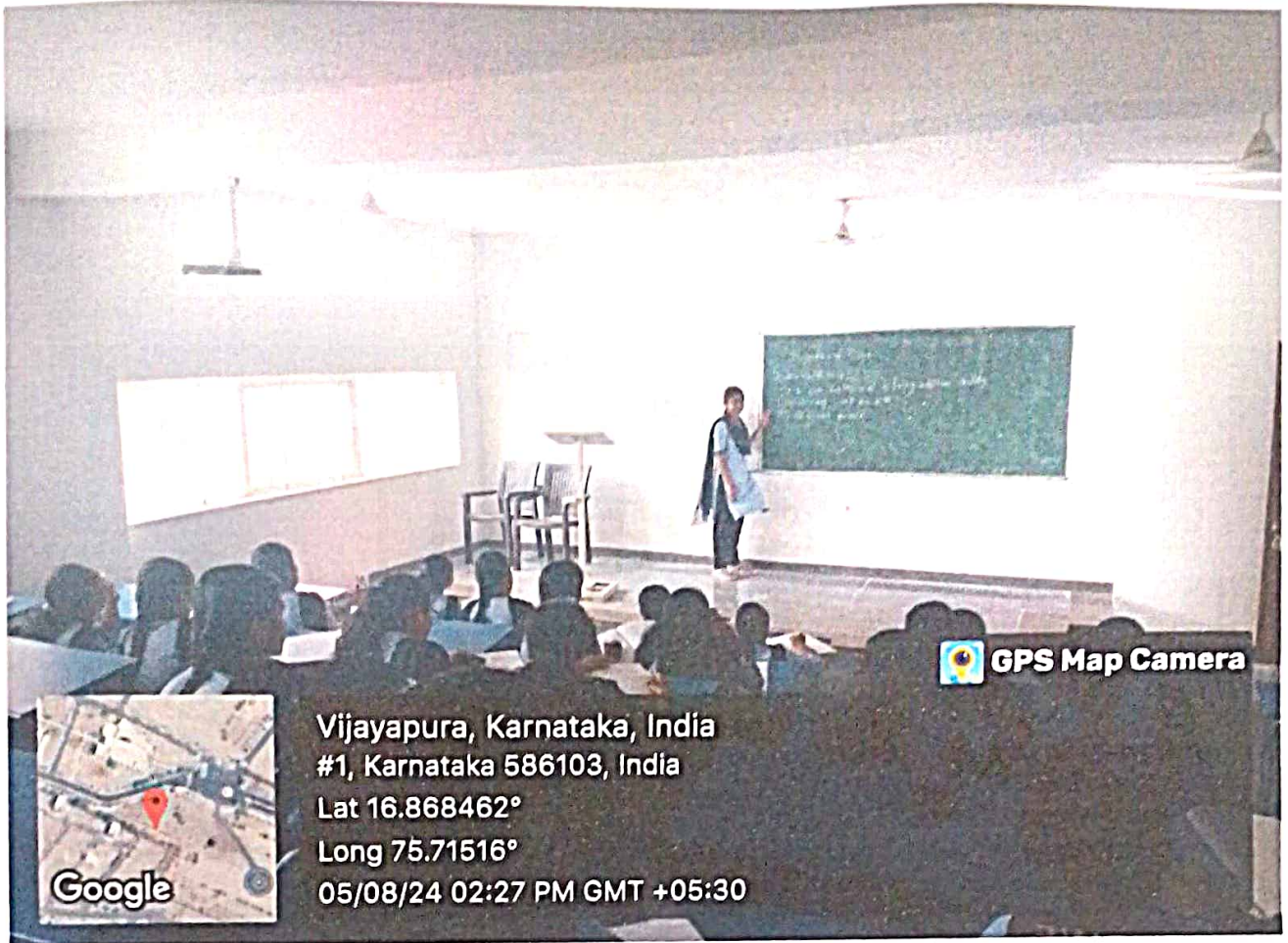
$$= \frac{1}{0.5} [-0.1125 - 0.0012 + 0.0203]$$

$$= \frac{1}{0.5} [-0.0934]$$

$$f'(1.5) = -0.1868$$









B. L. D. E. ASSOCIATION'S
S. B. ARTS AND K. C. P. SCIENCE COLLEGE
VIJAYAPUR
DEPARTMENT OF MATHEMATICS



Date: 08/08/2024

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6.	U15KM23S0099	Vishnu K	II	Vishnu. B.K
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9.	U15KM23S0168	Vilasmati Biradar	II	Vilasmati
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12.	U15KM23S0209	Rukmini Darveshi	II	Rukmini
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18.	U15KM23S0250	Soumya Lakkanagaon	II	Soumya
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21.	U15KM23S0302	Kaveri Hullur	II	Kaveri
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23.	U15KM23S0349	Aishwarya Asangihal	II	Aishwarya
24.	U15KM23S0358	Lakshmi Patil	II	Patil
25.	U15KM23S0359	Satish Pujari	II	Satish
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31.	U15KM22S0311	Amruta Shiroom	IV	Amruta
32.	U15KM22S0312	Revati Nesur	IV	Amruta S
33.	U15JU22S0058	Prema Bagalkot	IV	Prema
34.	U15KM22S0221	Abdul Yadagiri	IV	Abdul

35.	U15KM22S0222	Shrishail Pujari	IV	Shrishail
36.	U15KM22S0224	Sahana Sajjan	IV	Sahana
37.	U15KM22S0225	Sheetal Kadamani	IV	Sheetal
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45.	U15KM22S0162	Akshata Badagi	IV	A.R. Badagi
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48.	U15KM22S0158	Shweta Hunasagi	IV	Shweta
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50.	U15KM22S0125	Sahana Mallappagol	IV	Sahana
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52.	U15KM22S0108	Shailashree Kanashetti	IV	Pagad
53.	U15KM22S0102	Swati Chavhan	IV	Swati
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55.	U15KM22S0024	Madhumati Kalagond	IV	Madhumati
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93.	U15KM21S0163	Prajwal Aihole	VI	Prajwal
94.	U15KM21S0173	Aishwarya Umarani	VI	Aishwarya
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