B.Sc III rd Sem
Ist Internal Test 2023-2024 [odd Sem]

B. L. D. E. Association's
S. B. ARTS & K. C. P. SCIENCE COLLEGE, VIJAYAPUR

I INTERNAL TEST 2023-2024

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Q No-I

$\frac{30}{30}$

a) $(4x+3y+1)dx + (3x+2y+1)dy = 0$. be given
which is of the form $Mdx + Ndy = 0$.
where $M = 4x+3y+1$ $N = 3x+2y+1$.

$\frac{\partial M}{\partial y} = 3$ $\frac{\partial N}{\partial x} = 3$.

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$\therefore$ hence the given exquation is exact differential equation.

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b) $(D^2-9)y = 0$.
given $(D^2-9)y=0$
Auxiliary equation (A.E) $\Rightarrow m^2-9=0$.
$m^2 = 9$.
$m = \pm 3$

$\therefore m_1 = 3$ $m_2 = -3$.
$\therefore$ The roots are real & distinct.
Then the General Solution is
$y = C_1e^{3x} + C_2e^{-3x}$.

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c) Osillatory sequence :- A sequence $\{a_n\}$ is said to be osill-atory sequence if it neither converges to finite number nor diverges to $\pm\infty$. is called as osillatory sequence.
ex:- $\sum_{n=0}^{\infty}(-1)^n$. is a example. $\lim_{n\to\infty}(-1)^n$ $S_n = 1-1+1-1+ ... \pm(-1)^n$
$S_n = 0$ when n is even & $S_n=1$ when n->odd

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d) $1^2+2^2+3^2+ ... + n^2 + ...$
let $\sum_{n=1}^{\infty} U_n = \Sigma 1^2+2^2+3^2+ ... + n^2 + ...$
S_n is nth partial sum of ΣU_n.
$S_n = U_1 + U_2 + U_3 + ... + U_n$ $U_1 = 1^2$ $U_2 = 2^2$ $U_3 = 3^2$ $U_n = n^2$
$= 1^2+2^2+3^2+ ... + n^2$.
$S_n = n^2$
taking limit on both side
$\lim_{n\to\infty} S_n = \lim_{n\to\infty} n^2$
$= \infty^2$
$= \infty$.
$\therefore$ the series diverges to $+\infty$.

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Q no 2)

a) Necessary condition:-

Given that $Mdx + Ndy = 0$ be a exact differential equation, we have to prove that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

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let $Mdx + Ndy = 0$ be exact — (1).

by the definition of exact differential equation

$$dv = M\,dx + N\,dy. \quad —(2)$$

If $v = f(x,y)$ is a function of two variables x & y such that total derivative of v is given by

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy. \quad (3)$$

by comparing the coefficients of dx & dy.

$$M = \frac{\partial v}{\partial x} \qquad N = \frac{\partial v}{\partial y}.$$

$$\frac{\partial M}{\partial y} = \frac{\partial^2 v}{\partial x \partial y} \qquad \cancel{\frac{\partial N}{\partial y}} \; \frac{\partial N}{\partial x} = \frac{\partial^2 v}{\partial x \partial y}.$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

Sufficient condition:-

let $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ be given then we have to prove that $Mdx + Ndy = 0$ is exact.

let, $\int M\,dx = v$. (Integration is done by treating y as constant.)

Consider $\int M dx = V$.

differentiating partially wrt x.

$$\frac{\partial}{\partial x}\int M dx = \frac{\partial v}{\partial x}.$$

$$M = \frac{\partial v}{\partial x}. \quad —(4)$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial^2 v}{\partial x \partial y}.$$

we have $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

$$\therefore \frac{\partial N}{\partial x} = \frac{\partial^2 v}{\partial x \partial y}. \quad —(5)$$

Integrating on both side w.r.t. x by keeping y as a constant.

$$\int \frac{\partial N}{\partial x}\, dx = \int \frac{\partial^2 v}{\partial x \partial y}\, dx.$$

$$N = \frac{\partial v}{\partial y} + \text{function of } y$$

$$= \cancel{\partial y}$$

$$N = \frac{\partial v}{\partial y} + f(y). \quad —⑥.$$

by eqn ~~⑤~~ ④ & ⑥. we get

$$M\,dx + N\,dy = \frac{\partial v}{\partial x}\, dx + \left(\frac{\partial v}{\partial y} + f(y)\right) dy.$$

$$= \frac{\partial v}{\partial x}\, dx + \frac{\partial v}{\partial y}\, dy + f(y)\, dy.$$

$$= \frac{\partial v}{\partial x}\, dx + \frac{\partial v}{\partial y}\, dy + f(y)\, dy$$

$$M\,dx + N\,dy = dv + f(y)\, dy \qquad (\because \text{by eqn } ③).$$

$$= dv + d\int f(y)\, dy$$

$$M\,dx + N\,dy = d\left(v + \int f(y)\, dy\right).$$

$\therefore$ $M\,dx + N\,dy$ is exact differential equation.

c) $\frac{1}{f(D)} \cdot e^{ax} = \frac{1}{f(a)} \cdot e^{ax}$, if $f(a) \neq 0$.

proof:- by successive differentiation. we have

$$D e^{ax} = a e^{ax}$$
$$D^2 e^{ax} = a^2 e^{ax}$$
$$D^3 e^{ax} = a^3 e^{ax}$$
$$\vdots$$
$$D^n e^{ax} = a^n e^{ax}.$$

We observe that RHS is obtained by replacing D by a.

$\therefore$ ~~$f(D)\, e^{ax}$~~ $f(D) \cdot e^{ax} = f(a) \cdot e^{ax}$.

by dividing $\frac{1}{f(D)}$ on both side.

Qn

we get

$$\frac{1}{f(D)} \cdot f(D) \cdot e^{ax} = \frac{1}{f(D)} \cdot f(a) \cdot e^{ax}.$$

$$e^{ax} = \frac{1}{f(D)} \cdot f(a) \cdot e^{ax}.$$

rearranging the terms

$$\frac{1}{f(D)} \cdot e^{ax} = \frac{1}{f(a)} \cdot e^{ax}.$$

$$\Rightarrow \frac{1}{f(D)} \cdot e^{ax} = \frac{1}{f(a)} \cdot e^{ax} \quad \text{if } f(a) \neq 0.$$

d) $(D^3 + D^2 + 4D + 4)y = 0.$

Given $(D^3 + D^2 + 4D + 4)\, y = 0.$

Auxiliary equation is $m^3 + m^2 + 4m + 4 = 0.$

$$m^3 + m^2 + 4m + 4 = 0.$$

take $m = -1$, $(-1)^3 + (-1)^2 + 4(-1) + 4 = 0$

$$-1 + 1 - 4 + 4 = 0$$

$$0 = 0.$$

$\Rightarrow m = -1$ is a root by inspection method.

$\therefore (m+1)$ is a factor.

then by synthetic division method.

$m=-1$	1	1	4	4
	$\downarrow$	-1	0	-4
	1	0	4	$\boxed{0}$

$\therefore (m+1)\,(m^2 + 0m + 4) = 0.$

$(m+1)\,(m^2+4) = 0.$

$m = -1$, $\quad m^2 = -4$

$m = \sqrt{-4} = 2\sqrt{-1} = 2i$

$m = -1$, $\quad m = 0 \pm 2i.$ ($p \pm qi$)

$\therefore$ The one root is real & two roots are complex.

$\therefore$ The general solution is,

$$y = c_1 e^{-x} + e^{0x}\,[c_2 \cos 2x + c_3 \sin 2x]$$

If $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$. PT $\lim_{n\to\infty} a_n + b_n = a+b$.

proof:- Given that a_n & b_n be the two sequences such that

$$\lim_{n\to\infty} a_n = a \quad -(1) \qquad \lim_{n\to\infty} b_n = b \quad -(2)$$

by the definition of limit of a sequence for $\epsilon > 0$ $\exists$ a positive integer m_1, m_2 such that

$$|a_n - a| < \frac{\epsilon}{2} \quad \forall n \geq m_1 \quad -(3)$$

$$|b_n - b| < \frac{\epsilon}{2} \quad \forall n \geq m_2 \quad -(4).$$

let $M = \max\{m_1, m_2\}$.

then $|a_n - a| < \frac{\epsilon}{2}$ $\quad |b_n - b| < \frac{\epsilon}{2}$ $\quad \forall n \geq m$. (5).

Consider $|(a_n + b_n) - (a+b)| = |a_n + b_n - a - b|$

$= |a_n - a + b_n - b|$

$\leq |a_n - a| + |b_n - b|$ $(\because |x+y| \leq |x| + |y|)$

$< \frac{\epsilon}{2} + \frac{\epsilon}{2}$.

$< \epsilon$.

$\therefore |(a_n + b_n) - (a+b)| < \epsilon$. $\quad \forall n \geq m$.

$\therefore$ the sequence $\{a_n + b_n\}$ is convergent.

$$\lim_{n\to\infty} (a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n,$$

$$\lim_{n\to\infty} (a_n + b_n) = a + b.$$

hence proved.

) P-series:- The series $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p}$. is called as p-series

P series. $\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p}$.

i) convergent if $P > 1$

ii) divergent if $P \leq 1$

proof:-

Case(i) when $P > 1$.

$$\sum \frac{1}{n^P} = \frac{1}{1^P} + \frac{1}{2^P} + \frac{1}{3^P} + \dots + \frac{1}{n^P}.$$

$$\frac{1}{1^P} = 1 \quad , \quad \frac{1}{2^P} + \frac{1}{3^P} < \frac{1}{2^P} + \frac{1}{2^P}.$$

$$< \frac{2}{2^P}.$$

$$\frac{1}{2^P} + \frac{1}{3^P} < \frac{1}{2^{P-1}}$$

$$\frac{1}{4^P} + \frac{1}{5^P} + \frac{1}{6^P} + \frac{1}{7^P} < \frac{1}{4^P} + \frac{1}{4^P} + \frac{1}{4^P} + \frac{1}{4^P}.$$

$$< \frac{4}{4^P}.$$

$$< \frac{1}{4^{P-1}}$$

$$< \frac{1}{(2^{P-1})^2}$$

$$\frac{1}{8^P} + \frac{1}{9^P} + \dots + \frac{1}{15^P} < \frac{1}{8^P} + \frac{1}{8^P} + \frac{1}{8^P} + \frac{1}{8^P} + \frac{1}{8^P} + \frac{1}{8^P} + \frac{1}{8^P} + \frac{1}{8^P}$$

$$< \frac{8}{8^P}$$

$$< \frac{1}{8^{P-1}}$$

$$< \frac{1}{(2^{P-1})^3},$$

$$\therefore \sum \frac{1}{n^P} = \frac{1}{1^P} + \frac{1}{2^P} + \frac{1}{3^P} + \frac{1}{4^P} + \frac{1}{5^P} + \frac{1}{6^P} + \frac{1}{7^8} + \frac{1}{8^P} + \dots + \frac{1}{n^P}.$$

$$\sum \frac{1}{n^P} < 1 + \frac{1}{2^{P-1}} + \frac{1}{(2^{P-1})^2} + \frac{1}{(2^{P-1})^3} + \dots$$

The above series is geometric Series is & having common ratio $r = \frac{1}{2^{P-1}} < 1$, if $P > 1$

$$\therefore \quad S_n = 1 + \frac{1}{2^{P-1}} + \frac{1}{(2^{P-1})^2} + \frac{1}{(2^{P-1})^3} + \dots$$

$$S_n = \frac{1 - (1/2^{P-1})^n}{1 - 1/2^{P-1}} \qquad \left[\because S_n = \frac{1-r^n}{1-r}\right.$$

taking limit on b.s.

$$\therefore \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1-(1/2^{p-1})^n}{1-1/2^{p-1}}$$

$$= \lim_{n\to\infty} \frac{1}{1-1/2^{p-1}} - \lim_{n\to\infty} \frac{(1/2^{p-1})^n}{1-1/2^{p-1}}$$

$$= \frac{1}{1-1/2^{p-1}} - 0 \quad \left[\because p<1, \ \frac{1}{2}<1 \quad \frac{1}{2^\infty}=0.\right.$$

$$\therefore \lim_{n\to\infty} S_n = \frac{1}{1-1/2^{p-1}}$$

which is finite.

~~The sequence S_n is Convergent.~~

The geometric series is convergent.

hence $\sum \frac{1}{n^p}$ is also convergent. if $p>1$.

case ii) $p \le 1$.

a) $p=1$

$$\sum \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p}$$

$$\sum \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}.$$

we have $1=1$ $\quad \frac{1}{2}=\frac{1}{2}.$

$$\frac{1}{3}+\frac{1}{4} > \frac{1}{4}+\frac{1}{4}$$

$$> \frac{2}{4}$$

$$\frac{1}{3}+\frac{1}{4} > \frac{1}{2}$$

$$\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8} > \frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$$

$$> \frac{4}{8}$$

$$> \frac{1}{2}.$$

$$\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\cdots+\frac{1}{16} > \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\cdots+\frac{1}{16}.$$

$$> \frac{8}{16}$$

$$> 1/2.$$

$$\sum \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}.$$

$$> 1 + \frac{1}{2} + \frac{1}{2} + \dots$$

After the 1st term the series is geometric series having common ratio 1.

$\therefore$ The geometric series is ~~convergent~~. divergent

hence the $\sum \frac{1}{n^p}$ is divergent. if $p=1$

b) $p<1$.

$n^p < n$

$\frac{1}{n^p} > \frac{1}{n}$

$\sum \frac{1}{n^p} > \sum \frac{1}{n}$.

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} > \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}.$$

The terms of the $\sum \frac{1}{n^p}$ is greater than ~~that of the~~ that of corresponding terms $\sum \frac{1}{n}$.

by the case (ii) a.

$\sum \frac{1}{n}$ is divergent.

hence $\sum \frac{1}{n^p}$ is divergent if $p<1$

d) $\frac{1}{3 \cdot 7} + \frac{1}{4 \cdot 9} + \frac{1}{5 \cdot 11} + \frac{1}{6 \cdot 13}$.

$$\sum u_n = \sum \frac{1}{(n+2)(2n+5)}$$

$$\therefore u_n = \frac{1}{(n+2)(2n+5)} = \frac{1}{n^2(1+2/n)(2+5/n)}$$

$$v_n = \frac{1}{n^2}.$$

by the Comparison test

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{\frac{1}{n^2(1+2/n)(2+5/n)}}{1/n^2}$$

$$= \lim_{n\to\infty} \frac{1}{n^2(1+2/n)(2+5/n)} \times \frac{n^2}{1}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2}.$$

$\therefore$ which is finite & non zero.

Σu_n & Σv_n Converges or diverges together.

Consider $\Sigma v_n = \Sigma \frac{1}{n^2}$.

which is in the form of $\Sigma \frac{1}{n^p}$ where $p = 2 > 1$.

by p-series test. when $P \geq 1$.

Σv_n is convergent.

by comparison test

Σu_n is also convergent.

$\lim_{n\to\infty} a_n = l$, $\lim_{n\to\infty} b_n = m$ then $\lim_{n\to\infty} a_n b_n = lm$.

proof:- The given sequence $\{a_n\}$ & $\{b_n\}$ are convergent then they are bounded $\exists$ a real no. k & K such that $|a_n| \leq k \ \forall n$

$|b_n| \leq K \ \forall n$.

Consider $|a_n b_n - lm| = |a_n b_n - b_n l + b_n l - lm|$

$= |b_n(a_n - l) + l(b_n - m)|$

$\leq |b_n(a_n - l)| + |l(b_n - m)|$

$\leq |b_n| |a_n - l| + |l| |b_n - m|$.

$\leq |K| |a_n - l| + |l| |b_n - m|$ — (1).

given $\lim_{n\to\infty} a_n = l$ $\qquad \lim_{n\to\infty} b_n = m$.

by the definition of limit of sequence for $\epsilon > 0$ $\exists$ a real number m_1, m_2 such that

$|a_n - l| < \frac{\epsilon}{2|K|}$ $\quad \forall n \geq m_1$ — (2)

$|b_n - m| \leq \frac{\varepsilon}{2|l|}$ $\forall\ n \geq m_2$. — (3).

let $M = \max\{m_1, m_2\}$ such that
$|a_n - l| < \frac{\varepsilon}{2|K|}$, $|b_n - m| < \frac{\varepsilon}{2|l|}$ $\forall\ n \geq M$. — (4)

eqn. (4) in (1)

$$|a_n b_n - lm| \leq |K|\,|a_n - l| + |l|\,|b_n - m|$$
$$\leq |K| \cdot \frac{\varepsilon}{2|K|} + |l| \frac{\varepsilon}{2|l|}$$
$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$$
$$|a_n b_n - lm| < \varepsilon. \quad \forall\ n \geq M.$$

therefore $\{a_n b_n\}$ is convergent
$\therefore \lim_{n\to\infty} a_n b_n = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$

$\lim_{n\to\infty} a_n b_n = l.m$

hence proved.

Q.no 2

b) $(3x^2 + 6xy^2)\,dx + (6x^2y + 4y^3)\,dy = 0.$
Given equation is in the form of $M\,dx + N\,dy = 0$,

where $M = 3x^2 + 6xy^2$ $\quad N = 6x^2y + 4y^3$
$\frac{\partial M}{\partial y} = 12xy$ $\quad \frac{\partial N}{\partial x} = 12xy$.

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

hence the given equation is exact.

Then the General Solution is
$\int_{y\text{-constant}} M\,dx + \int N$ (not containing x terms) $dy = 0$

$\Rightarrow \int (3x^2+6xy^2)\,dx + \int 4y^3\,dy = 0.$

$\Rightarrow \frac{3x^3}{3} + \frac{6x^2y^2}{2} + \frac{4y^4}{4} = c.$

$\Rightarrow x^3 + 3x^2y^2 + y^4 = c.$

2

$(4x+3y+1)dx + (3x+2y+1)dy = 0$

Given equation is in the form of $Mdx + Ndy = 0$.

where $M = 4x+3y+1$ $\quad$ $N = 3x+2y+1$.

$\frac{\partial M}{\partial y} = 3$ $\quad$ $\frac{\partial N}{\partial x} = 3.$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$

$\therefore$ Given equation is exact.

Then the general solution is

$\int M dx + \int N(\text{not containing } x \text{ terms})\, dy = 0$

$\int (4x+3y+1)\,dx + \int (2y+1)\,dy = 0$

$\Rightarrow \frac{4x^2}{2} + 3yx + x. + \frac{2y^2}{2} + y = c.$

$\Rightarrow 2x^2 + 3yx + x + y^2 + y = c$

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IInd INTERNAL TEST 2023 -2024

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1)

a) A equation in which dependent variables, ~~&~~ independent variable & ~~their~~ differential co-efficients of dependent variables with respect to independent variables are involved such equation is known as Differential Equation. 2

b) Given $p = \log(y-px)$

$\Rightarrow e^p = y - px$

$y = px + e^p$

This is Clairaut's equation.

$\therefore$ General Solution is $y = cx + e^c$ 2

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c) Any differential equation of form $y = px + f(p)$ is known as Clairauts Equation.

Ex: $y = px + e^p$ 2

d) Given $(D^2 - 2D + 3)y = 0$

AE: $m^2 - 2m + 3 = 0$

$m = \dfrac{-(-2) \pm \sqrt{4 - 4(1)3}}{2(1)}$

$= \dfrac{2 \pm \sqrt{4-12}}{2} = \dfrac{2 \pm \sqrt{-8}}{2} = \dfrac{2 \pm 2\sqrt{2}i}{2}$ 2

$\Rightarrow m = 1 \pm i\sqrt{2}$

Hence, $y = e^x (C_1 \cos\sqrt{2}x + C_2 \sin\sqrt{2}x)$

I - 6 7

II - 8

III - 8

IV -

2) Statement.

a) Any equation of form $Mdx + Ndy = 0$ — (*) is exact differential equation if $\partial M/\partial y = \partial N/\partial x$.

To prove: $Mdx + Ndy = 0$ is sufficient condition for $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Proof: Let $\int M dx = V$ where V is $f(x,y)$.

Integrating w.r.t x holding y as constant;

$$M = \frac{\partial V}{\partial x} \quad \text{— (1)}$$

$$\frac{\partial M}{\partial y} = \frac{\partial^2 V}{\partial x \partial y}$$

from given wkT $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{\partial^2 V}{\partial x \partial y}$

$$\frac{\partial N}{\partial x} = \frac{\partial^2 V}{\partial x \partial y}$$

Integrating w.r.t x holding y as constant.

$$N = \frac{\partial V}{\partial y} + f(y) \quad \text{— (2)}$$

Substitute (1) & (2) in (*)

$$Mdx + Ndy = \frac{\partial V}{\partial x} dx + \left[\frac{\partial V}{\partial y} + f(y)\right] dy$$

$$= \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + f(y)\, dy$$

$$= dV + f(y) dy \quad \left[\because dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy\right]$$

$$= d\{V + \int f(y) dy\}.$$

Hence, Sufficient Condition is proved.

Given $xy\,dx - (x^2+2y^2)\,dy = 0$ — (1) is of form $M\,dx + N\,dy = 0$

$\Rightarrow M = xy$ $\qquad N = -x^2 - 2y^2$

$\frac{\partial M}{\partial y} = x$ $\qquad \frac{\partial N}{\partial x} = -2x$

as $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, eqⁿ (1) is not exact.

$$f(y) = \frac{\partial N/\partial x - \partial M/\partial y}{M}$$

$$= \frac{-2x - x}{xy} = \frac{-3x}{xy} = \frac{-3}{y}$$

$\therefore$ IF $= e^{\int f(y)\,dy}$

$$= e^{\int -3/y\,dy} = e^{-3\log y} = y^{-3} = 1/y^3$$

Multiply IF to eqⁿ (1)

$$\frac{1}{y^3}(xy)\,dx - \frac{1}{y^3}(x^2+2y^2)\,dy = 0$$

$$\frac{x}{y^2}\,dx - \frac{x^2}{y^3} - \frac{2}{y}\,dy = 0.$$

It is of form $M\,dx + N\,dy = 0$.

$\therefore M = \frac{x}{y^2}$ $\qquad N = \left(-\frac{x^2}{y^3} - \frac{2}{y}\right)$

$\frac{\partial M}{\partial y} = \frac{-2x}{y^3}$ $\qquad \frac{\partial N}{\partial x} = \frac{-2x}{y^3}$

as $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ eqⁿ is exact.

Thus General Solution is $\int M\,dx + \int N\,dy = 0$

$\Rightarrow \int \frac{x}{y^2}\,dx + \int \frac{-2}{y}\,dy = 0$

$\Rightarrow \frac{x^2}{2y^2} - 2\log y = C$ is the General Solution.

3) b) Given $y = 2px + p^4x^2$ — ①

$$\frac{dy}{dx} = 2\left[p + x\frac{dp}{dx}\right] + 2xp^4 + x^2 4p^3\frac{dp}{dx}$$

$$p = 2p + 2x\frac{dp}{dx} + 2xp^4 + 4x^2p^3\frac{dp}{dx}$$

$$2p - p + 2xp^4 + 2x\frac{dp}{dx}\left[1 + 2xp^3\right] = 0$$

$$p + 2xp^4 + 2x\frac{dp}{dx}\left[1 + 2xp^3\right] = 0.$$

$$p\left[1+2xp^3\right] + 2x\frac{dp}{dx}\left[1+2xp^3\right] = 0$$

$$\left[1+2xp^3\right] + \left[p + 2x\frac{dp}{dx}\right] = 0$$

Neglecting term not containing dp/dx.

$$p + 2x\frac{dp}{dx} = 0$$

$$\frac{dx}{x} + \frac{2dp}{p} = 0.$$

Integrate $\log x + 2\log p = \log c$

$$\log x + \log p^2 = \log c$$

$$\log xp^2 = \log c$$

$$xp^2 = c$$

$$p = \sqrt{c/x} = c/\sqrt{x}$$

$y = 2px + p^4x^2 \Rightarrow y - p^4x^2 = 2px$ — ②

Put p value in eqⁿ ②

$$y - \frac{c^4}{x^2}x^2 = 2\frac{c}{\sqrt{x}}x$$

$$(y - c^4)^2 = 2\frac{c^2}{x}x^2$$

$(y-c)^2 = 2cx$ // is the General Solution

Given $p = \sin(y - px)$ — ①

$$\sin^{-1} p = y - px$$

$$y = px + \sin^{-1} p \quad \text{— ②}$$

It is Clairauts' Equation ∴ put $p = c$

$$\boxed{y = cx + \sin^{-1} c \quad \text{is the general solution.}}$$

Diff ② w.r.t. p

$$0 = x + \frac{1}{\sqrt{1-p^2}}$$

$$\cancel{\frac{1}{\sqrt{1-p^2}} = -x}$$

$$\cancel{\sqrt{1-p^2} = -1/x}$$

$$\cancel{1-p^2 = \sqrt{\frac{-1}{x}}}$$

$$\cancel{p^2 = 1 + \sqrt{\frac{-1}{x}}}$$

$$x\sqrt{1-p^2} + 1 = 0.$$

$$\sqrt{1-p^2} = -\frac{1}{x}$$

$$1 - p^2 = \left(\frac{-1}{x}\right)^2$$

$$1 - p^2 = \frac{1}{x^2}$$

$$\therefore p^2 = 1 + \frac{1}{x^2}$$

$$p = \sqrt{1 + \frac{1}{x^2}}$$

Put p value in ②

$y = \sqrt{1+\frac{1}{x^2}}\,(x) + \sin^{-1}\sqrt{1+\frac{1}{x^2}}$ is the singular solution

8) c) Given. Successive Differentiation of $\sin(ax+b)$

$D(\sin(ax+b)) = a\cos(ax+b)$

$D^2\sin(ax+b) = -a^2\sin(ax+b)$.

$(D^2)^1\sin(ax+b) = (-a^2)^1\sin(ax+b)$ — ①

$D^3\sin(ax+b) = (a)^3\cos(ax+b)$

$D^4\sin(ax+b) = (-a)^4\sin(ax+b)$

$(D^2)^2\sin(ax+b) = (-a^2)^2\sin(ax+b)$ — ②

⋮

$(D^2)^n\sin(ax+b) = (-a^2)^n\sin(ax+b)$ — ③

From ①, ② & ③ we get:

$D^2\sin(ax+b) = (-a^2)\sin(ax+b)$.

$f(D^2)\sin(ax+b) = f(-a^2)\sin(ax+b)$

Operating $1/f(D^2)$

$$\frac{1}{f(D^2)} f(D^2)\sin(ax+b) = \frac{1}{f(D^2)} f(-a^2)\sin(ax+b)$$

$$\sin(ax+b) = \frac{f(-a^2)}{f(D^2)}\sin(ax+b)$$

Hence, $\frac{1}{f(D^2)}\sin(ax+b) = \frac{1}{f(-a^2)}\sin(ax+b)$

provided $f(-a^2)$

) ~~Succes Let e^{ax} has a~~

b) Let U be a f^n of x
And Successive Differentiation of $e^{ax}U$ is

$$D(e^{ax}U) = e^{ax}D(U) + U\,D(e^{ax})$$
$$= e^{ax}D(U) + aU e^{ax}$$
$$= e^{ax}(D+a)U \quad -(1)$$

$$D^2(e^{ax}U) = D\left[e^{ax}(D+a)U\right]$$
$$= e^{ax}D\left[(D+a)U\right] + (D+a)U\left[D(e^{ax})\right]$$
$$= e^{ax}(D^2+D)U + (D+1)U e^{ax}$$
$$= e^{ax}\left[D^2+D+D+a\right]U$$
$$= e^{ax}\left[D^2+2D+a\right]U$$
$$= e^{ax}\left[D+a\right]^2 U \quad -(2)$$

lll^{ly} $D^3(e^{ax}U) = e^{ax}[D+a]^3U$

$\vdots$

$$D^n(e^{ax}U) = e^{ax}[D+a]^n U \quad -(3)$$

From (1), (2) & (3) we get.

$$D(e^{ax}U) = e^{ax}(D+a)^n U$$
$$f(D)\,e^{ax}U = e^{ax}f(D+a)\cdot U$$

~~Let $e^{ax}U = V \Rightarrow U = \frac{V}{e^{ax}}$ Operating $f(D)$ on both side~~

~~$f(D)U \Rightarrow U = \frac{V}{f(D)}$~~ $\cancel{f(D)e^{ax}U\left(\frac{1}{f(D)}\right) = e^{ax}f(D+a)U\left(\frac{1}{f(D)}\right)}$

$$\cancel{e^{ax}U = \frac{e^{ax}f(D)U}{f(D+a)}}$$

~~Let $f(D)U = V \Rightarrow U = \frac{V}{f(D)}$~~

$$\cancel{\frac{e^{ax}V}{f(D)} = e^{ax}\frac{f(D+a)}{f(D)}\frac{V}{f(D)} = e^{ax}}$$

Let $f(D)U = V$

$\Rightarrow U = \dfrac{V}{f(D)}$

$$f(D)\, e^{ax} \frac{V}{\cancel{f(D)}} = e^{ax} f(D+a) \frac{V}{f(D)}$$

$$\frac{1}{f(D+a)} e^{ax} V = \frac{1}{f(D)} e^{ax} V$$

Hence $\dfrac{1}{f(D)} e^{ax} V = \dfrac{1}{f(D+a)} e^{ax} V$ where V is $f(x)$

B SC IV sem I[st] Internal Exam

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30/30

Qno - I

a) $\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = 0$

given $\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = 0$.

It can be written as replace $D = \frac{\partial}{\partial x}$ & $D' = \frac{\partial}{\partial y}$.

$D^2 z + 6DD'z + 9D'^2 z = 0$.

$\therefore (D^2 + 6DD' + 9D'^2)z = 0$.

replace $D = m$ & $D' = 1$ then A.E is

$m^2 + 6m + 9 = 0$.

$m^2 + 3m + 3m + 9 = 0$.

$m(m+3) + 3(m+3) = 0$

$(m+3)(m+3) = 0$

$m = -3, -3$.

The roots are real and repeat.

Then $C.F = \phi_1(y-3x) + x\phi_2(y-3x)$

which is general solution.

2

b) $z = ax + by + ab$.

given $z = ax + by + ab$ — ①.

differentiate partially w.r.t x.

$\frac{\partial z}{\partial x} = a$.

$p = a$. — ② $\quad (\because \frac{\partial z}{\partial x} = p)$

differentiate partially w.r.t y.

$\frac{\partial z}{\partial y} = b$.

$q = b$ — ③. $\quad (\because \frac{\partial z}{\partial y} = q)$

2

Substitute eqn ② & ③ in ①.

$z = px + py + pq$.

c) $L[t^3+3t^2-6t+8]$

$L[t^3+3t^2-6t+8] = L[t^3]+3L[t^2]-6L[t]+L[8]$

$$= \frac{3!}{s^{3+1}} + 3\times\frac{2!}{s^{2+1}} - 6\times\frac{1!}{s^{1+1}} + \frac{8}{s}.$$

2

$$= \frac{6}{s^4} + \frac{3\times 2}{s^3} - \frac{6}{s^2} + \frac{8}{s}.$$

$$= \cancel{\frac{6+6s-6s^2+8s^3}{s^4}}$$

$\therefore L[t^3+3t^2-6t+8] = \frac{8s^3-6s^2+6s+6}{s^4}.$

QNO-2

a) $(D^3-D^2D'-8DD'^2+12D'^3)z=0.$

Given $(D^3-D^2D'-8DD'^2+12D'^3)z=0$

Replace $D=m$ & $D'=1$. then A.E is

$m^3-m^2-8m+12=0.$

$m=-3$, $0=0$.

$(-3)^3$ $27-9-124+12$

$\therefore (m+3)$ is a first factor.

$m=-3$ is root by inspection method.

~~Then~~ by synthetic division method

$m=-3$	1	-1	-8	12
	↓	-3	12	-12
	1	-4	4	0

$\therefore (m+3)(m^2-4m+4)=0$

$(m+3)=0$, $m^2-4m+4=0$

$m=-3$. $m^2-2m-2m+4=0$

$m(m-2)-2(m-2)=0$

$(m-2)(m-2)=0$

$m=2,2$.

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$\therefore m=-3,2,2.$

Then roots are real & d
Then
$CF = \phi_1(y+2x) + x\phi_2(y+2x) + \phi_3(y-3x)$
which is a general solution of eqn.

$(D^2 + 2DD' + D'^2)z = e^{2x+3y}$.
Given $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$.
Replace $D = m$ & $D' = 1$ then
AE: $m^2 + 2m + 1 = 0$.
$m^2 + m + m + 1 = 0$
$m(m+1) + 1(m+1) = 0$
$(m+1)(m+1) = 0$.
$m = -1, -1$

The roots are real & repeat.
$C.F = \phi_1(y-x) + x\phi_2(y-x)$
Now $P.I = \dfrac{1}{D^2+2DD'+D'^2} e^{2x+3y}$.

$F(D,D') = D^2 + 2DD' + D'^2$
$F(2,3) = 2^2 + 2\times2\times3 + 3^2$ ($a = 2$, $b = 3$)
$= 4 + 12 + 9$
$= 25 \neq 0$
$F(a,b) \neq 0$.
$V = 2x+3y$

$= \dfrac{1}{25}\iint e^V \, dv\, dv$
$= \dfrac{1}{25}\int e^V dv$
$= \dfrac{1}{25} e^V$.

$P.I = \dfrac{1}{25} e^{2x+3y}$.

Then the general solution is $Z = CF + PI$.

$Z = \phi_1(y-x) + x\phi_2(y-x) + \dfrac{1}{25} e^{2x+3y}$.

c) Theorem :- TO Obtain the partial differential equation of the form $Pp + Qq = R$ by eliminating arbitrary constant function ϕ from $\phi(u,v) = 0$.

Proof :- let $u = u(x,y,z)$ & $v = v(x,y,z)$ be any two functions of x, y, z connected with each others through arbitrary relation of type $\phi(u, v) = 0$. where ϕ is arbitrary function

$$\phi(u, v) = 0 \quad \text{--- (1)}$$

differentiate partially w.r.t x.

$$\frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial x}\right) + \frac{\partial \phi}{\partial v}\left(\frac{\partial v}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial x}\right) = 0.$$

$$\frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial x} + 0 + \frac{\partial u}{\partial z}p\right) + \frac{\partial \phi}{\partial v}\left(\frac{\partial v}{\partial x} + 0 + \frac{\partial v}{\partial z}p\right) = 0.$$

$$\frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial x} + p\frac{\partial u}{\partial z}\right) = -\frac{\partial \phi}{\partial v}\left(\frac{\partial v}{\partial x} + p\frac{\partial v}{\partial z}\right) \quad \text{(2)}$$

differentiate partially w.r.t y.

$$\frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial y}\right) + \frac{\partial \phi}{\partial v}\left(\frac{\partial v}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial y} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial y}\right) = 0$$

$$\frac{\partial \phi}{\partial u}\left(0 + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}q\right) + \frac{\partial \phi}{\partial v}\left(0 + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}q\right) = 0$$

$$\frac{\partial \phi}{\partial u}\left(\frac{\partial u}{\partial y} + q\frac{\partial u}{\partial z}\right) = -\frac{\partial \phi}{\partial v}\left(\frac{\partial v}{\partial y} + q\left(\frac{\partial v}{\partial z}\right)\right) \quad \text{(3)}$$

dividing eqn (2) by (3)
we get

$$\frac{\frac{\partial u}{\partial x} + p\frac{\partial u}{\partial z}}{\frac{\partial u}{\partial y} + q\frac{\partial u}{\partial z}} = \frac{\frac{\partial v}{\partial x} + p\frac{\partial v}{\partial z}}{\frac{\partial v}{\partial y} + q\frac{\partial v}{\partial z}}$$

$$\left(\frac{\partial u}{\partial x}+p\frac{\partial u}{\partial z}\right)\left(\frac{\partial v}{\partial y}+q\frac{\partial v}{\partial z}\right)=\left(\frac{\partial v}{\partial x}+p\frac{\partial v}{\partial z}\right)\left(\frac{\partial u}{\partial y}+q\frac{\partial u}{\partial z}\right)$$

$$\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+q\frac{\partial u}{\partial x}\frac{\partial v}{\partial z}+p\frac{\partial u}{\partial z}\frac{\partial v}{\partial y}+pq\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}=$$

$$\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+q\frac{\partial u}{\partial z}\frac{\partial v}{\partial x}+p\frac{\partial u}{\partial y}\frac{\partial v}{\partial z}+pq\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}$$

$$\Rightarrow p\left(\frac{\partial u}{\partial z}\frac{\partial v}{\partial y}-\frac{\partial u}{\partial y}\frac{\partial v}{\partial z}\right)+q\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial u}{\partial z}\frac{\partial v}{\partial x}\right)$$

$$=\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}-\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}$$

which is of the form of $Pp+Qq=R$.

where $P=\frac{\partial u}{\partial z}\frac{\partial v}{\partial y}-\frac{\partial u}{\partial y}\frac{\partial v}{\partial z}$

$$Q=\frac{\partial u}{\partial x}\frac{\partial v}{\partial z}-\frac{\partial u}{\partial z}\frac{\partial v}{\partial x}$$

$$R=\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}-\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}.$$

hence eqn is partial differential eqn of 1st Order & 1st degree. It shows that $\phi(u,v)=0$ is general solution of $Pp+Qq=R$.

Q:-3

a) Periodic function:- If $f(x)$ is said to be a periodic function of if $f(x+T)=f(x)$ for it ~~Some~~ is true for some value T & for all x. then for positive value of T for which $f(x+T)=f(x)$ is true then it is called periodic function.

ex:- $\sin(t+\pi)=\sin t$ with period π

4

Theorem:-

If $f(x)$ is a periodic function of period T. i.e $f(t+T)=f(t)$. then

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt.$$

proof :- $f(x)$ is periodic function.

by the definition of laplace transform

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^T e^{-st} f(t)\,dt + \int_T^\infty e^{-st} f(t)\,dt.$$

In second integral part take $t = u+T$

$\therefore du = dt.$

$t = T \qquad u = 0$

$t = \infty \qquad u = \infty.$

$$L[f(t)] = \int_0^T e^{-st} f(t)\,dt + \int_0^\infty e^{-s(u+T)} f(u+T)\,du$$

$$L[f(t)] = \int_0^T e^{-st} f(t)\,dt + \int_0^\infty e^{-su} e^{-sT} f(u+T)\,du$$

$$= \int_0^T e^{-st} f(t)\,dt + e^{-sT} \int_0^\infty e^{-su} f(u+T)\,du$$

by the defn of periodic function.

$$f(u+T) = f(u)$$

$$= \int_0^T e^{-st} f(t)\,dt + e^{-sT} \int_0^\infty e^{-su} f(u)\,du$$

In second integral part replace u by t

$$= \int_0^T e^{-st} f(t)\,dt + e^{-sT} \int_0^\infty e^{-st} f(t)\,dt.$$

$$= \int_0^T e^{-st} f(t)\,dt + e^{-sT} L[f(t)]$$

(by defn of laplace transform)

$$L[f(t)] - e^{-sT} L[f(t)] = \int_0^T e^{-st} f(t)\,dt$$

$$L[f(t)][1 - e^{-sT}] = \int_0^T e^{-st} f(t)\,dt$$

$$L[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt \quad //$$

hence proved.

2) If $L[f(t)] = F(s)$ & $g(t) = \begin{cases} f(t-a) & \text{if } t > a \\ 0 & \text{if } t < a \end{cases}$

then $L[g(t)] = e^{-as} F(s)$

proof :- given $L[f(t)] = F(s)$

by the definition of Laplace transform

$$L[g(t)] = \int_0^\infty e^{-st} g(t)\,dt$$

$$= \int_0^a e^{-st} \times 0 \times dt + \int_a^\infty e^{-st} f(t-a)\,dt$$

$$= 0 + \int_a^\infty e^{-st} f(t-a)\,dt$$

$$= \int_a^\infty e^{-st} f(t-a)\,dt$$

take $t-a = y$. then $dt = dy$.

if $t = a$ $\quad y = 0$

$t = \infty$ $\quad y = \infty$.

$$\therefore L[g(t)] = \int_0^\infty e^{-s(y+a)} f(y)\,dy$$

$$= \int_0^\infty e^{-sy} \cdot e^{-sa} f(y)\,dy$$

$$= e^{-sa} \int_0^\infty e^{-sy} f(y)\,dy$$

$$L[g(t)] = e^{-sa} F(s)$$ [by definition of Laplace transform]

hence proved

c) $f(x) = x^{2} - 1$. $-\pi < x < \pi$.

Given $f(x) = x - 1$.

$f(-x) = -x - 1 \neq f(x) \neq -f(x)$

This function is neither even nor odd.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx . \quad \text{—(1)}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{\pi}\int_{-\pi}^{\pi} (x-1)\, dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} - x\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left\{\left[\frac{\pi^2}{2} - \pi\right] - \left[\frac{\pi^2}{2} - (-\pi)\right]\right\}$$

$$= \frac{1}{\pi}\left\{\frac{\pi^2}{2} - \pi - \frac{\pi^2}{2} - \pi\right\} = \frac{1}{\pi}(-2\pi) = -2 ,$$

$\therefore a_0 = -2$,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx = \frac{1}{\pi}\int_{-\pi}^{\pi} (x-1) \cos nx\, dx$$

$$= \frac{1}{\pi}\left\{(x-1)\frac{\sin nx}{n} - (1)\left(\frac{-\cos nx}{n^2}\right)\right\}_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left\{(x-1)\frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right\}_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left\{\left[(\pi-1)\frac{\sin n\pi}{n} + \frac{\cos n\pi}{n^2}\right] - \left[(\pi-1)\frac{\sin n(-\pi)}{n} + \frac{\cos n(-\pi)}{n^2}\right]\right.$$

$$= \frac{1}{\pi}\left\{\left[0 + \frac{\cos n\pi}{n^2}\right] - \left[(-\pi-1)\frac{(-\sin n\pi)}{n} + \frac{\cos n\pi}{n^2}\right]\right\}$$

$$= \frac{1}{\pi}\left\{\frac{\cos n\pi}{n^2} - \left[0 + \frac{\cos n\pi}{n^2}\right]\right.$$

$$= \frac{1}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos n\pi}{n^2}\right]$$

$= 0$.

B.SC IV Sem IInd Internal Exam

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Class: B.Sc IV Sem
Sub: Mathematics
Ro.No: 119

I – 6
II – 12
III – 12

30/30

Q.NO 1:

a) $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$

$r + 4s + 4t = 0$, $S = 4$ $R = 1$ $T = 4$

$S^2 - 4RT = 4^2 - 4\times1\times4$

$= 16 - 16$

$= 0$

This equation is parabolic

b) $p^2 - q^2 = 1$ — ①

$z = ax + by + c$ — ②

diff par w.r.t. x and y

$\frac{\partial z}{\partial x} = a$, $\frac{\partial z}{\partial y} = b$

$\boxed{p = a}$ $\boxed{q = b}$

$z = a^2$ substitute the $p = a$, $q = b$ in eq ①

$a^2 - b^2 = 1$

$b^2 = a^2 - 1$

$b = \pm\sqrt{a^2 - 1}$ — ③

eq ③ is substitute eq ②

$\boxed{z = ax + \pm(\sqrt{a^2 - 1})y + c}$

c) $L[t e^{at}]$

The periodic func

$f(x+T) = f(x)$

$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[f(s)]$

$L[f(e^{at})] = \frac{1}{s-a} = F(s)$

$L[t f(t)] = (-1)^1 \frac{d}{ds}\left(\frac{1}{s-a}\right)$

$\boxed{L[t\, f(t)] = \frac{1}{(s-a)^2}}$

d).

$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$

$b_n = \frac{2}{l}\int_0^l 1 \sin n\pi x \, dx$

$b_n = -\frac{\cos n\pi x/l}{n\pi/l}\Big]_0^l$

$b_n =$

d) The finite sine transform of function f is interval (0,l) of is called The finite sine transform

$F_S(n) = \int_0^l f(x) \sin \frac{n\pi x}{l} dx$

n is positive integer.

2

Q.No 2).

a). $r + 2s + t = 0$

$R = 1 \quad S = 2 \quad T = 1$

$S^2 - 4RT = 2^2 - 4 \times 1 \times 1$

$= 4 - 4 = 0$

4

This equation is parabolic equation

The characteristic of the equation

The quadratic equation $R\lambda^2 + S\lambda + T = 0$

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda^2 + \lambda + \lambda + 1 = 0$

$\lambda(\lambda+1) + 1(\lambda+1) = 0$

$(\lambda+1)(\lambda+1) = 0$

$\lambda = -1, \quad \lambda = -1$

The characteristic equation

$\frac{dy}{dx} + \lambda_1 = 0 \qquad \frac{dy}{dx} + \lambda_2 = 0$

$\frac{dy}{dx} - 1 = 0$

integration

$y - x = c_1 \qquad \boxed{c_1 = x - y}$

$u = x - y \qquad v = x + y$

The Jacobian propity

$$J\frac{(u.v)}{(x.y)} = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1+1 = 2 \neq 0$$

$u = x - y \qquad v = x + y$

$$p = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x}$$

$$= \frac{\partial z}{\partial u}(1) + \frac{\partial z}{\partial v}(1)$$

$$q = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y}$$

$$= (-)\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}(1)$$

$$r = \frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$= \left(\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right)\left(\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}\right)$$

$$= \frac{\partial^2 z}{\partial u^2} + 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2}$$

$$s = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$= \left(\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right)\left(-\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}\right)$$

$$= -\frac{\partial^2 z}{\partial u^2} - \cancel{\frac{\partial^2 z}{\partial u \partial v}} + \cancel{\frac{\partial^2 z}{\partial u \partial v}} + \frac{\partial^2 z}{\partial v^2}$$

$$= -\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}$$

$$t = \frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \left(-\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right)\left(-\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}\right)$$

$$= +\frac{\partial^2 z}{\partial u^2} - \frac{\partial^2 z}{\partial u \partial v} - \frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2}$$

$$t = \frac{\partial^2 z}{\partial u^2} - 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2}$$

$$r + 2s + t = 0$$

$$\frac{\partial^2 z}{\partial u^2} + 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2} + 2\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} + \frac{\partial^2 z}{\partial u^2} - 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2}$$

$$\cancel{\frac{\partial^2 z}{\partial u \partial v}} \quad 4\frac{\partial^2 z}{\partial v^2} = 0$$

$$\boxed{\frac{\partial^2 z}{\partial v^2} = 0}$$

b). $u_t = c^2 u_{xx}$.

$u = XT \quad X = X(x) \quad T = T(t)$

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial (XT)}{\partial t} = c^2 \frac{\partial^2 (XT)}{\partial x^2}$$

$$X\frac{dT}{dt} = c^2 T \frac{d^2 X}{dx^2}$$

3

$$\frac{1}{X}\frac{d^2X}{dx} = \frac{1}{c^2 T}\frac{dT}{dt} = k.$$

$$\frac{d^2X}{dx} = kX \qquad \frac{dT}{dt} = kc^2 T$$

$$\frac{d^2X}{dx} - kX = 0 \qquad \frac{dT}{dt} - kc^2T = 0$$

$m^2 = 0 \quad \boxed{m = 0, 0} \qquad m = 0$

Then the equation $-k = 0$ of completes

$X = (C_1 + C_2 x) e^{0x} \qquad T = C_3 e^{0t}$

Then the equation of the

$u = XT$

$u = (C_1 + C_2 x) e^{0x} C_3 e^{0t}$

$u = (C_1 + C_2 x) C_3$

$x = (c_1 c_3 + c_2 c_3 x)$ $A = c_1 c_3$ $B = c_2 c_3 x$

$x = A + Bx$

II type of the equation $k = +p^2$

$$\frac{d^2x}{dx^2} - p^2 = 0 \qquad \frac{dT}{dt} - c^2 p^2 T = 0$$

$m^2 = \pm p$ $m = +cp$

$x = (c_1'' + c_2 x) e^{-p}$ $T = c_3'' e^{-cpt}$

III type of the equation $k = -p^2$

$$\frac{d^2x}{dx^2} + p^2 = 0 \qquad \frac{dT}{dt} + c^2 p^2 T = 0$$

$m = \pm ip$ $m = -c^2p^2$

complete solution

$x = (c_1'' \cos px + c_2'' \sin px)\, c_3'' e^{-c^2p^2t}$

$x = A \cos px\, e^{-p^2c^2t} + B \sin px\, e^{-p^2ct}$

eg. $z = px + qy + 3(pq)^{1/3}$ —①

$z = px + qy + f(pq)$ —②

$p = a$ $q = b$

$z = ax + by + 3(ab)^{1/3}$

diff par wrt a.

$0 = x + (ab)^{-2/3} b$

$x = -(ab)^{-2/3} b$ —③

diff partial wrt b.

$0 = y + 1(a.b)^{-2/3} a$

$y = -(a.b)^{-2/3} a$ —④

equations mult on both

$ax = -a^{-2/3}\, b^{-2/3}, b \times a$

$ax = -a^{1/3}\, b^{1/3}$

$ax = -(a.b)^{1/3}$ — (5) $= -1/xy$

The equation mult (1) eq (2) b

$by = -(a.b)^{-2/3}\, a.b = -a^{-2/3}a, b^{2/3}$

$by = -a^{\frac{1}{3}}\, b^{1/3}$

$by = -(ab)^{1/3} = -1/xy$

$ax = -(a.b)^{1/3}$ | $by = -(a,b)^{1/3}$

3 cube on both side.

$(ax)^3 = (-(a.b)^{1/3})^3$ $(by)^3 = (-(a,b)^{1/3})^3$

$a^3x^3 = -a.b$ $(by)^3 = -a.b$

$x^3 = -\frac{ab}{a^3}$ $y^3 = -\frac{ab}{b^3}$

$x^3 = -\frac{b}{a^2}$ $y^3 = -\frac{a}{b^2}$

$x^3y^3 = +\frac{ba}{a^2b^2} = \frac{1}{ab}$

$(xy)^3 = \frac{1}{ab}$

1/3 ration on both side

$((xy)^3)^{1/3} = \frac{1}{(ab)^{1/3}}$

$xy = \frac{1}{(ab)^{1/3}}$

$(ab)^{1/3} = \frac{1}{xy}$

$z = ax + by + 3(a.b)^{+1/3}$

$z = -1/xy - 1/xy + \frac{3}{\ }$ $= -2$ $+3$

$$z = \frac{1}{xy} = \boxed{xyz = 1}$$

$$xyz = 1$$

$p^3 + q^3 = 3pqz$

$f(z, p, q) = 0$

III type of the each other

$$p = \frac{dz}{dx} \quad q = a\frac{dz}{dx}$$

$$\left(\frac{dz}{dx}\right)^3 + a^3\left(\frac{dz}{dx}\right)^3 = 3\frac{dz}{dx}a\frac{dz}{dx}z$$

$$\left(\frac{dz}{dx}\right)^3\left[1 + a^3\right] = 3az\left(\frac{dz}{dx}\right)^2$$

$$\frac{dz}{dx}\left[1+a^3\right] = 3az$$

$$\frac{dz}{z} = \frac{3a}{1+a^3}dx$$

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integrate both side

$$\log z = \frac{3a}{1+a^3}x + C$$

$$\log z = \left[\frac{3a}{1+a^3}\right](x+ay) + C$$

Q.NO 3].

a] If $L[f(t)] = F(s)$ then $L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$

4 — By the definition of laplace transform.

Q.No 3]

a] $L[f(t)] = F(s)$ then

$L\left[\frac{f(t)}{t}\right] = \int_s^\infty F(s)\,ds$

By the definition of Laplace transform

Let $F(s) = \int_0^\infty e^{-st} f(t)\,dt$

integrating s to ∞ in both side

$$\int_s^\infty F(s)\,ds = \int_s^\infty \int_0^\infty e^{-st} f(t)\,dt\,ds$$

$$\int_s^\infty F(s)\,ds = \int_0^\infty \left[\int_s^\infty e^{-st}\,ds\right] f(t)\,dt$$

$$= \int_0^\infty \frac{e^{-st}}{t} f(t)\,dt$$

$$= \frac{L[f(t)]}{t}$$

$$\boxed{\frac{L[f(t)]}{t} = \int_s^\infty F(s)\,ds}$$ //

b] $L[f(t)] = F(s)$

$L[U(t-a)] = \frac{e^{-as}}{s}$

By the unit step function.

By the definition of L.T.

$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$U(t-a) = \begin{cases} 0 & 0 < t < a \\ 1 & t > a \end{cases}$

$L[U(t-a)] = \int_0^a e^{-st}\,U(t-a)\,dt + \int_a^\infty e^{-st}\,U(t-a$

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$$L[U(t-a)] = \int_0^a e^{-st} \cdot 0\, dt + \int_a^\infty e^{-st} \cdot 1\, dt$$

$$= 0 + \left[\frac{e^{-st}}{-s}\right]_a^\infty$$

$$= \frac{1}{-s}\left[e^{-\infty} - e^{-as}\right]$$

$$= \frac{1}{-s}\left[0 - e^{-as}\right]$$

$$= \frac{e^{-as}}{s} - 0$$

$$\boxed{L[U(t-a)] = \frac{e^{-as}}{s}}$$

d). The finite fourier cosine transform $f(x) = 2-x$ in $(0,2)$.

$$F_c(n) = \int_0^l f(x) \cos \frac{n\pi x}{l}\, dx \qquad l = 2$$

$$F_c(n) = \int_0^2 (2-x) \cos n\pi x/2\, dx.$$

$$F_c(n) = \int_0^2 \left[(2-x) \frac{\sin n\pi x/2}{n\pi/2} - (-1)\frac{(-\cos n\pi x/2)}{n^2\pi^2/4}\right]$$

$$= \left[(2-2)\frac{\sin n\pi 2/2}{n\pi/2} - (-1)\frac{(-\cos n\pi 2/2)}{n^2\pi^2/4}\right]_0^2 - \left[(2-0)\frac{\sin n\pi 0}{n\pi/2} - \frac{\cos n\pi 0}{n^2\pi^2/4}\right]$$

$$= \left[0 - \frac{4}{n^2\pi^2}(-1)^n\right] - \left[0 - \frac{4}{n^2\pi^2}\right]$$

$$= \frac{4}{n^2\pi^2}\left[1 - (-1)^n\right]$$

$$F_c(n) = \frac{4}{n^2\pi^2}\left[1 - (-1)^n\right]$$

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c) $f(x) = (\pi - x)$ interval $(0, \pi)$

half range $\sin \frac{n\pi x}{l}$

$f(x) = \frac{2}{l}\int_0^l f(x)\, b_n$

$f(x) = 2\sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$ $\quad l = \pi$.

$f(x) = b_n = \frac{2}{\pi}\int_0^{\pi} (\pi - x)\sin nx\, dx$

$b_n = \frac{2}{\pi}\left[(\pi - x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$

$b_n = \frac{2}{\pi}\left[0 - 0\right] - \left[\frac{-\pi + 0}{n}\right]$

$= \frac{2}{\pi} \times \frac{\pi}{n} = \frac{2}{n}$

$b_n = 2/n$

$(\pi - x) = \sum_{n=1}^{\infty} \frac{2}{n} \sin \frac{n\pi x}{l}$

d) The half range cosine series

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$ $\quad l = \pi$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

$a_0 = \frac{2}{\pi}\int_0^{\pi} (\pi - x)\, dx$

$= \frac{2}{\pi}\left[\pi x - \frac{x^2}{2}\right] = \frac{2}{\pi}\left[\pi^2 - \frac{\pi^2}{2}\right] = \frac{2}{\pi} \times$

$\boxed{a_0 = \pi}$

$a_n = \frac{2}{\pi}\int_0^{\pi} (\pi - x)\cos nx\, dx.$

$a_n = \frac{2}{\pi}\left[(\pi - x)\frac{\sin nx}{n} - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$

$a_n = \frac{2}{\pi}\left[0 - \frac{(-1)^n}{n^2}\right] - \left[0 - \frac{1}{n^2}\right]$

$= -\frac{2}{\pi n^2}\left[(-1)^n + 1\right] \quad a_n = \frac{2}{\pi n^2}\left[1 - (-1)^n\right]$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$

$(\pi - x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{l}\left[1 - (-1)^n\right].$

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