

**B.L.D.E. Association's
S.B. ARTS AND K.C.P. SCIENCE COLLEGE
VIJAYAPUR**



DEPARTMENT OF MATHEMATICS

**PROJECT REPORT
2023-24**

BLDEA's S.B. Arts and K.C.P. Science College, Vijayapur
Department of Mathematics
BSc VI Semester
Students Project List 2023-24

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3	U15KM21S0020	AKASH VIJAYAKUMAR			
4	U15KM21S0023	UMA SHIVANAND			
5	U15KM21S0039	PARVATI NEMAGOUD	2	Dr. Manjunath Jyoti	Series Solution of Differential Equations
6	U15KM21S0059	VIDYASHREE B PATIL			
7	U15KM21S0066	KAVERI RAVI			
8	U15KM21S0081	KAVYA BUDIHAL			
9	U15KM21S0090	YUVARAJ MANTAYYA	3	Miss. Rajashree Katte	A Study on the Sphere
10	U15KM21S0320	Megha Dodamani			
11	U15KM21S0098	APOORVA AWATI			
12	U15KM21S0114	MAHESHWARI	4	Miss. Rajashree Katte	Vector Spaces
13	U15KM21S0116	POOJA BASAVARAJ			
14	U15KM21S0130	NIKHITA SIDDESHWAR			
15	U15KM21S0136	AISHWARYA PATIL			
16	U15KM21S0141	PRAMOD BHIMARAY	5	Smt. Suprita V. Pattanashatti	A Study on Set Theory
17	U15KM21S0154	BHAGYASHREE			
18	U15KM21S0156	VINAYAK DATTATRAY			
19	U15KM21S0169	SHRIKANT NIGADI			
20	U15KM21S0186	RANI BIRADAR	6	Smt. Suprita V. Pattanashatti	A Study on Matrices
21	U15KM21S0213	SAHANA THOKE			
22	U15KM21S0216	VISHWANATH K			
23	U15KM21S0229	SIDDARTH ASHOK			
24	U15KM21S0258	PALLAVI BASAVARAJ	7	Dr. Veena R. Desai	A Study on Spectra of Graphs
25	U15KM21S0270	AKASH SINDOOR			
26	U15KM21S0285	PRANOTI PRALHAD			
27	U15KM21S0286	SHILPA MANJUNATH			
28	U15KM21S0288	SADHANA	8	Dr. Veena R. Desai	A Study on Graph Transformation
29	U15KM21S0289	ANNARAY BHAVIKATTI			
30	U15KM21S0414	AKASH NANAGOUDA			
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33	U15KM21S0001	KRUPA MAHADEV	9	Dr. S. N. Patil	Applications of Laplace transform in Engineernig
34	U15KM21S0007	RUCHITA			
35	U15KM21S0008	AKSHATA BIRAPPA			
36	U15KM21S0010	SAMPATAKUMAR			
37	U15KM21S0011	AISHWARYA	10	Dr. S. N. Patil	Calculus Variation and its Applications
38	U15KM21S0012	ASHWINI RATHOD			
39	U15KM21S0017	LINGARAJ SANGAPPA			
40	U15KM21S0026	SABALE DIKSHEETA			
41	U15KM21S0382	Metre Aishwarya			

42	U15KM21S0036	SIDDANNA MALLANNA	11	Sri. Anand Jirli	Continuity and its Applications
43	U15KM21S0040	SAIPRAKASH BEKINAL			
44	U15KM21S0043	NIKITA BIRADAR			
45	U15KM21S0057	ABHISHEK BASAVARAJ			
46	U15KM21S0062	MANIKANT	12	Sri. Anand Jirli	Differntiability
47	U15KM21S0072	SWAPNA KANASE			
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49	U15KM21S0094	PUSHPA SHIVAPPA			
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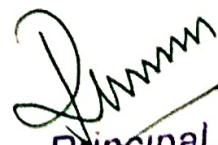


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LINEAR TRANSFORMATIONS AND ITS MATRIX



B. L. D. E. Association's
S. B. Arts and K. C. P. Science College
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DEPARTMENT OF MATHEMATICS

A Disseration Submitted To The
RANICHANNAMMA UNIVERSITY, BELAGAVI

for the partial fulfillment for the award of

Bachelor of Science

By

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PROJECT GUIDE

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August – 2024



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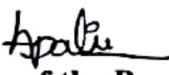


DEPARTMENT OF MATHEMATICS

Date: 13/08/2024

CERTIFICATE

This is to certify that the project entitled “Linear Transformations and its Matrix” carried out by ---**Mr.Sudeep D Rathod(U15KM21S0096), Miss.Bhuvaneshwari A Agasar(U15KM21S0097), Miss.Akshata S Pujari(U15KM21S0124), Miss.Mangala R Tulajanavar(U15KM21S0120)** are studying in B.Sc. VI semester during the year 2023-24, has completed the project. This Project work is in partial fulfillment for the award of degree of Bachelor of Science. The project work satisfies the requirements prescribed in the curriculum of “**Rani Channamma University, Belagavi**”.


Guide of the Project


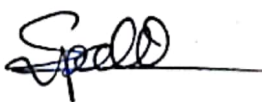
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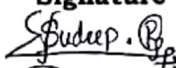

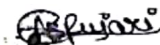
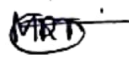

Principal,
S.B. Arts & K.C.P. Sc. College,
Vijayapur.

Signature of Examiner


1. 
2. 

DECLARATION

We hereby declare that the entire work embodied in this project entitled "Linear Transformations and its Matrix" submitted to Department of Mathematics for the partial fulfillment for the award of Bachelor of Science, is the result of investigation carried out by us in the Department of Mathematics under the supervision and guidance of **Miss. Laxmi Patil**, lecturer, Department of Mathematics, S. B. Arts and K. C. P. Science College, Vijayapur. This project has not been submitted for any diploma or degree.

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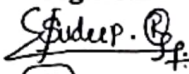



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Write on your own (by Students)

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CHAPTER 1

LINEAR TRASFORMATIONS

1.1 .INTRODUCTION

Linear Transformation is a function from one vector space to another vector space satisfying certain conditions.

In particular linear transformation from R^n to R^m is known as the Euclidean linear transformation .linear transformation have important applications in physics, Engineering and various branches of mathematics.

Definition

Let v and w be two vector spaces

Function $T: V \rightarrow W$ is called a linear transformation from v to w if for all u,u and v and all scalars K,

1.2.Properties of linear transformation

If $T: U \rightarrow V$ is linear transformation, then

01. $T(0) = 0^1$ where o and 0^1 are zero vectors of u and v respectively

$$T(\alpha + 0) = T(\alpha) + T(0)$$

$$T(\alpha) = T(\alpha) + T(0)$$

$$T(\alpha) + 0^1 = T(\alpha) + T(0)$$

$$T(0) = 0^1$$

02 consider

$$T[\alpha + (-\alpha)] = T(\alpha) + T(-\alpha)$$

$$T(0) = T(\alpha) + T(-\alpha)$$

$$0^1 = T(\alpha) + T(-\alpha)$$

Linear Transformation and it's Matrix

$T(-\alpha)$ is the inverse of $T(\alpha)$

$$\therefore T(-\alpha) = -T(\alpha)$$

(03) We shall prove the result by mathematical induction

Let $p(n): T(c_1 \alpha_1 + c_2 \alpha_2 + \dots + c_n \alpha_n)$

Now

$P(1): T(c_1 \alpha_1) = c_1 T(\alpha_1)$ as T is linear

As T is linear $P(1)$ is true

Hence the result is true for $n=1$

Let us assume that $p(k)$ is true for same +ve integer k .

I.e, $T(c_1 \alpha_1 + c_2 \alpha_2 + \dots + c_k \alpha_k)$

$$= c_1 T(\alpha_1) + c_2 T(\alpha_2) + \dots + c_k T(\alpha_k) \text{ is true}$$

Now we shall show that $p(k+1)$ is true,

Consider,

$$T(c_1 \alpha_1 + c_2 \alpha_2 + \dots + c_k \alpha_k + c_{k+1} \alpha_{k+1}) = 0$$

$$\begin{aligned} & T(c_1 \alpha_1 + c_2 \alpha_2 + \dots + c_k \alpha_k) + T(c_{k+1} \alpha_{k+1}) = \\ & c_1 T(\alpha_1) + c_2 T(\alpha_2) + \dots + c_k T(\alpha_k) + c_{k+1} T(\alpha_{k+1}) \end{aligned}$$

(Since $P(k)$ is true and T is linear

$\therefore P(k+1)$ is true

$\therefore P(k) = P(k+1)$ thus by induction $P(n)$ is true for all integral values of n .

Since the following properties asserts that a linear transformation T can be completely determined by it's values on the elements of a basis of the domain space.

Linear Transformation and it's Matrix

1.2.1. Standard Definition Linear Transformations

Matrix transformation: Let $T: R_n \rightarrow R_m$ be a linear transformation then there always exist an $m \times n$ matrix A such that

$$T(X) = AX$$

This transformation is called the matrix transformation or the Euclidean linear transformation. Here A is called the standard matrix for T . It is denoted by $[T]$

For example $T: R_3 \rightarrow R_2$ defined by

$$T(x, y, z) = (x = y - z, 2y = 3z, 3x + 2y + 5z)$$
 is a matrix transformation.

Zero Transformation: Let V and W be vector spaces. The mapping $T: V \rightarrow W$ be defined by $T(u) = 0 \forall u \in V$. T is called the zero transformation.

It is easy to verify that T is linear transformation.

Identity Transformation: Let V be any vector space. The mapping $L: V \rightarrow V$

Defined by $L(u) = u \forall u \in V$ is called the Identity Transformation on V

It is for the reader to verify that L is linear

1.2.2. Linear Transformation from images of basic vectors

A Linear Transformation is completely defined by the images of any set of basis vectors. Let $T: V \rightarrow W$ be a linear transformation and (v_1, v_2, \dots, v_n) can be any basis for V . Then the image $T(v)$ of any vector $u \in V$ can be calculated using following steps

Step 01

Express u as a linear combination of the basis vectors (v_1, v_2, \dots, v_n) say

Linear Transformation and it's Matrix

$$V = k_1 v_1 + k_2 v_2 + \dots + k_n v_n$$

Step 02

Apply the linear transformation T on v as

$$T(V) = T(k_1 v_1 + k_2 v_2 + \dots + k_n v_n)$$

$$T(v) = k_1 T(v_1) + k_2 T(v_2) + \dots + k_n T(v_n)$$

1.2.3. Composition of Linear Transformation

Let $T_1: u \rightarrow v$ and $T_2: v \rightarrow w$ be a linear transformation. Then the condition of T_2 with T_1 is the linear transformation defined by,

$$(T_2 \circ T_1)(u) = T_2(T_1(u)), \text{ where } u \in U$$

Suppose that $T_1: R^n \rightarrow R^m$ and $T_2: R^m \rightarrow R^k$ are linear transformation. Then there exist matrices A and B of order $m \times n$ and $k \times m$ respectively

Such that

$$T_1(x) = A(x) \text{ and } T_2(x) = B(x)$$

$$\text{Thus } A = [T_1] \text{ and } B = [T_2]$$

$$((T_2 \circ T_1)(x)) = T_2(T_1(x)) = T_2(Ax) = B(Ax) = (BA)x = ([T_1][T_2])(x)$$

So we have

$$T_2 \circ T_1 = [T_2][T_1]$$

Similarly, for 3 such linear transformations

$$T_3 \circ T_2 \circ T_1 = [T_3][T_2][T_1]$$

1.2.4. Theorem 01. Show that the mapping $f: v_3(R) \rightarrow v_2(R)$ defined by $f(a, b, c) = (c, a + b)$ is linear

Proof:→

Let $\alpha = (a_1, b_1, c_1)$ and $\beta = (a_2, b_2, c_2)$ be any two elements of $v_3(R)$

Now

$$\begin{aligned} f(\alpha + \beta) &= f\{(a_1, b_1, c_1) + (a_2, b_2, c_2)\} \\ &= f(a_1 + a_2, b_1 + b_2, c_1 + c_2) \\ &= \{(c_1 + c_2), (a_1 + a_2) + (b_1 + b_2)\} \\ &= \{(c_1, a_1 + b_1) + (c_2, a_2 + b_2)\} \\ &= f(a_1, b_1, c_1) + f(a_2, b_2, c_2) \\ &= f(\alpha) + f(\beta) \end{aligned}$$

1.2.5. Theorem 02. Show that the mapping $f: v_3(R) \rightarrow v_2(R)$ defined by $f(a, b, c) = (a - b, a + c)$ is linear

Proof:→

Here $f: v_3(R) \rightarrow v_2(R)$ defined as above is said to be linear, if

$$1. \quad f(\alpha + \beta) = f(\alpha) + f(\beta)$$

And $f(a\alpha) = af(\alpha), \forall a \in F, \alpha, \beta \in v_3(R)$

Let $\alpha = (a_1, a_2, a_3)$ and $\beta = (b_1, b_2, b_3)$ and $a \in F$

Now $f(\alpha + \beta) = f\{(a_1, a_2, a_3) + (b_1, b_2, b_3)\}$

$$\begin{aligned} &= f(a_1 + b_1, a_2 + b_2, a_3) \\ &= f[(a_1 + b_1) - (a_2 + b_2), (a_1 + b_1) + (a_3 + b_3)] \\ &= [(a_1 - a_2), (a_1 + a_3)] + [(b_1 - b_2), (b_1 + b_3)] \\ &= f(a_1, a_2, a_3) + f(b_1, b_2, b_3) = f(\alpha) + f(\beta) \end{aligned}$$

$$\therefore f(\alpha + \beta) = f(\alpha) + f(\beta)$$

Linear Transformation and it's Matrix

2. Also $f(a \alpha) = f[a(a_1, a_2, a_3)]$

$$=f(aa_1, aa_2, aa_3)$$

$$=(aa_1 - aa_2, aa_1 + aa_3)$$

$$=a(a_1 - a_2, a_1 + a_3)$$

$$=af(a_1, a_2, a_3) = af(\alpha)$$

DEFINITIONS

RANK OF LINEAR TRANSFORMATION $[r(T)]$:

Let $T: V \rightarrow W$ be a linear transformation the dimension of the range space $R(T)$ is called rank of linear transformation.

RANGE OF LINEAR TRRMATIANSFOON:

Let $T: V \rightarrow W$ be a linear transformation then range of T is the set $R(T) = \{T(\alpha); \alpha \in V\}$. $R(T)$ is the set of all T images of the elements of V therefore $R(T) \subseteq W$.

NULLITY OF LINEAR TRANSFORMATION:

Let $T: V \rightarrow W$ be a linear transformation the dimension of null space $N(T)$ is called as nullity of linear transformation.

NULL SPACE or KERNEL OF LINEAR TRANSFORMATION:

- Let $T: U \rightarrow V$ be a linear transformation. The null space (or kernel) of the subset of U consisting of all vectors u whose image under T is 0 and it is denoted by kernel (T) or $N(T)$.

Elementary operations do not change rank of the matrix.

$n \times n$ identity 1. Rank of the null matrix is not defined $= 0$

2. Rank of matrix $= n$

3. If A is $n \times n$ non singular matrix then $\rho(A) = \rho(A^{-1})\rho(A) = \rho(A^T) = \rho(AA^T) = \rho(A^{-1}) = \rho(A^0)$

4. $A^0 \rightarrow \text{Tranjugate} \rightarrow \text{Transposet conjugate}$.

5. Rank Row matrix is $\rho(A) = 0$ or 1

6. Rank Column matrix is $\rho(A) = 0$ or 1

Linear Transformation and its Matrix

1.3.RANK NULLITY THEOREM:

STATEMENT: Let $T: V \rightarrow W$ be a linear transformation and V be finite dimensional vector space then, $r(T) + n(T) = \dim(V)$

OR

Rank + nullity = dimension of domain

PROOF: Let V be a vector space of dimension m

I.e $\dim V = m$ and

$\dim(N(T)) = n = n(T)$

Since nullity of transformation is subspace V .

$$\therefore n < m$$

Let $B_1 = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be a basis of nullity of linear transformation $N(T)$.

We will extend this basis to basis of the vector space V .

$$B_2 = \{\alpha_1, \alpha_2, \dots, \alpha_n, \beta_1, \beta_2, \dots, \beta_s\}$$

$$n + s = m$$

Now, $T(\alpha_1), T(\alpha_2), \dots, T(\alpha_n)$ belongs to null space.

$$\therefore T(\alpha_1) = 0, T(\alpha_2) = 0, \dots, T(\alpha_n) = 0$$

$$(\alpha_i' \in N(T))$$

Let $S = \{T(\beta_1), T(\beta_2), \dots, T(\beta_s)\}$

We shall show that the set of S of s vectors of $R(T)$ is a basis of $R(T)$.

1. S spans $R(T)$

Since, B_2 is a basis of V .

\therefore The set $\{T(\alpha_1), T(\alpha_2), \dots, T(\alpha_n)\}$ and

Linear Transformation and its Matrix

$T(\beta_1), (\beta_2), \dots, \dots, T(\beta_s)$ Spans of $R(T)$.

But $T(\alpha_1) = 0, T(\alpha_2) = 0, \dots, \dots, T(\alpha_n) = 0$

Hence the set S spans $R(T)$.

2. S is linearly independent

Consider, $C_1 T(\beta_1) + C_2 T(\beta_2) + \dots + C_s T(\beta_s) = 0$

$$T(C_1 \beta_1 + C_2 \beta_2 + \dots + C_s \beta_s) = 0$$

$$C_1 \beta_1 + C_2 \beta_2 + \dots + C_s \beta_s = 0$$

$\therefore \exists$ Scalars $d_1, d_2, \dots, \dots, d_n$ such that

$$C_1 \beta_1 + C_2 \beta_2 + \dots + C_s \beta_s$$

$$= d_1 \alpha_1 + d_2 \alpha_2 + \dots + d_n \alpha_n$$

$$\Rightarrow C_1 \beta_1 + C_2 \beta_2 + \dots + C_s \beta_s - d_1 \alpha_1 - d_2 \alpha_2 - \dots - d_n \alpha_n$$

Since B_2 is basis of V

\therefore Vectors in B_2 are linearly independent.

$$\therefore d_1 = d_2 = \dots = d_n = C_1 = C_2 = \dots = C_s = 0$$

S is linearly independent

S is a basis of $R(T)$

$$\dim R(T) = s, \dim n(T) = n$$

$$n + s = m$$

$$\therefore n(T) + r(T) = \dim V$$

Linear Transformation and it's Matrix

1.3.1.THEOREM .Let $T: U \rightarrow V$ be a linear transformation then range of T
I.E $R(T)$ is subspace of T

Proof \rightarrow

Since $T(0) = 0$, '0' is in $R(T)$ and thus $R(T) \neq 0$

Let $v_1, v_2 \in R(T)$, Then $v_1 = T(u_1)$ and $T(u_2)$ for some u_1 and u_2 in U

NOW for $a, b \in F$ $au_1 + bu_2 = aT(u_1) + bT(u_2)$

$$= T(au_1 + bu_2) \in R(T)$$

Since $au_1 + bu_2 \in U$. Therefore $R(T)$ is subspace of V

Linear Transformation and it's Matrix

1.4.MATRIX OF LINEAR TRANSFORMATION

The matrix of a linear transformation is a matrix for which $T(\vec{x}) = A\vec{x}$, for a vector \vec{x} in the domain of T . this means that applying the transformation T to a vector is the same as multiplying by this matrix

Such a matrix can be found for any linear transformation T from R^n to R^m , for fixed value of n and m , and is unique to the transformation, in this lesson, we will focus on how exactly to find that matrix A , called the standard matrix for the transformation.

DEFINITION

Let $T: R_n \rightarrow R_m$ be a linear transformation. Then we can find a matrix such that $T(\vec{x}) = A\vec{x}$. In this case, we say that T is determined or induced by the matrix A is known as matrix of linear transformation.

1.4.1.PROPERTIES:HOW TO FIND THE MATRIX OF A LINEAR TRANSFORMATION

In order to find this matrix, we must first define a special set of vector from the domain called the standard basis. The big concept of a basis will be discussed when we look at general vector spaces, for now We just need to understand what vectors make up this set.

The standard basis for R^2 is:

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The standard basis for R^3 is:

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

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$$\vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

See the Pattern? We can define the standard basis like this for any R^n

The standard matrix of a transformation $T: R^n \rightarrow R^m$ has Column $T(\vec{e}_1), T(\vec{e}_2), \dots, T(\vec{e}_n)$, where $\vec{e}_1, \vec{e}_2, \dots, \vec{e}_n$ represent the standard basis that is:

$$T(\vec{x}) = A\vec{x} \leftrightarrow A = [T(\vec{e}_1) \ T(\vec{e}_2)]$$

Therefore, to find the standard matrix, we will find the image of each standard basis vectors. This is show in the following examples.

1.4.2.EXAMPLES

1: FIND THE STANDARD MATRIX FOR THE TRANSFORMATION T WHERE:

$$T\left(\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}\right) = \begin{bmatrix} x1 - x2 \\ 2x3 \end{bmatrix}$$

Solution:

T takes vectors with three entries to vectors with two entries therefore:

$$T: R^3 \rightarrow R^2$$

So the domain of T is R^3 .to find column of the standard matrix for the transformation we will need to find:

$$T(\vec{e}_1), T(\vec{e}_2), \text{ and } T(\vec{e}_3)$$

Using the given rule for T:

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$$T(\overrightarrow{e_1}) = T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 - 0 \\ 2(0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

1.4.3. FIND THE STANDARD MATRIX FOR THE TRANSFORMATION T WHERE:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ 2x_3 \end{bmatrix}$$

Solution:

T takes vectors with three entries to vectors with two entries therefore:

$$T: R^3 \rightarrow R^2$$

So the domain of T is R^3 . to find column of the standard matrix for the transformation we will need to find:

$$T(\overrightarrow{e_1}), T(\overrightarrow{e_2}), \text{ and } T(\overrightarrow{e_3})$$

Using the given rule for T:

$$T(\overrightarrow{e_1}) = T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 - 0 \\ 2(0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

1.4.4. THEOREM 01: EVERY MATRIX TRANSFORMATION IS A LINEAR TRANSFORMATION

Suppose that T is a matrix transformation such that $T(\vec{x}) = A\vec{x}$ for some matrix A and that the vectors \vec{u} and \vec{v} are in the domain. Then for arbitrary scalars c and d :

$$\begin{aligned}T(c\vec{u} + d\vec{v}) &= A(c\vec{u} + d\vec{v}) \\&= A(c\vec{u}) + A(d\vec{v}) \\&= cA\vec{u} + dA\vec{v} \\&= cT(\vec{u}) + dT(\vec{v})\end{aligned}$$

$$T(c\vec{u} + d\vec{v}) = cT(\vec{u}) + dT(\vec{v})$$

T must be a linear transformation

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1.5.APPLICATIONS

Matrix of linear transformation can be used to merge multiple transformations together into a single transformation

Linear transformation have numerous applications is various fields, including:

1. Linear Algebra: They are used to solve systems of linear equations, find eigenvalues and eigenvectors, and diagonalize matrices.
2. Computer Graphics: Linear transformations are used to perform rotations, scaling, and
3. Machine Learning: They are used in neural networks, principal component analysis (PCA), and singular value decomposition (SVD).
4. Physics and Engineering: Linear transformations describe the motion of the objects, including rotations, reflections, and projections.
5. Data Analysis: they are used in data compression, image
6. Differential Equations: Linear transformations help solve differential equations and analyze stability.
7. Markov Chains: They are used to model random processes and calculate steady-state probabilities.
8. Optimization: Linear transformations are used in Linea programming and optimization techniques.
9. signal Processing: They are used in filtering, convolution, and Fourier analysis.
10. Statistics: Linear transformations are used in filtering, hypothesis testing, confidence intervals, and regression analysis.

These applications leverage the power of linear transformations to simplify complex problems, reveal underlying structures, and facilitate efficient computations.

REFERENCE:

- 1) Linear algebra- by R.R. Mahajan, M.L. Bhave, V.G.Joshi.
- 2) Linear algebra second addition-Kenneth Hoffman,
Ray kunze.
- 3) First course in linear algebra- P. B .Bhattacharya,
S.K Jain, S.R. Nagpaul.
- 4) Modern algebra - A.R.Vasishtha, A.K. Vasishsta.

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CONCLUSION

A linear transformation is a mathematical function that maps vector from one vector space to another while preserving the operations of vector addition and scalar multiplication. The key conclusions about linear transformation are:

1. **Preservation of operations:** Linear transformations preserve the operations of vector addition and scalar multiplications
2. **Matrix representation:** Every linear transformation can be represented by a matrix.
3. **Composition:** The composition of two linear transformations is also a linear transformation.
4. **Inverse:** If a linear transformation is invertible, its inverse is also a linear transformation.
5. **Kernel and image:** Every linear transformation has a kernel (null space) and an image (range).
6. **Rank-nullity theorem:** The rank of a linear transformation plus the nullity equals the dimension of the domain.
7. **Invertability:** A linear transformation is invertible if and only if its matrix representation is invertible.

These conclusion from the foundation of linear algebra and are crucial for understanding many mathematical and real-world applications, such as data analysis, machine learning, and physics.