B.L.D.E. Association's
S.B. ARTS AND K.C.P. SCIENCE COLLEGE
VIJAYAPUR

DEPARTMENT OF MATHEMATICS

PROJECT REPORT
2023-24

BLDEA's S.B. Arts and K.C.P. Science College, Vijayapur
Department of Mathematics
BSc VI Semester
Students Project List 2023-24

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LINEAR TRANSFORMATIONS AND ITS MATRIX

B. L. D. E. Association's

S. B. Arts and K. C. P. Science College
Vijayapur - 586 103

DEPARTMENT OF MATHEMATICS

A Disseration Submitted To The

RANICHANNAMMA UNIVERSITY, BELAGAVI

for the partial fulfillment for the award of

Bachelor of Science

By

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CERTIFICATE

This is to certify that the project entitled "Linear Transformations and its Matrix" carried out by ---**Mr.Sudeep D Rathod(U15KM21S0096), Miss.Bhuvaneshwari A Agasar(U15KM21S0097), Miss.Akshata S Pujari(U15KM21S0124), Miss.Mangala R Tulajanavar(U15KM21S0120)** are studying in B.Sc. VI semester during the year 2023-24, has completed the project. This Project work is in partial fulfillment for the award of degree of Bachelor of Science. The project work satisfies the requirements prescribed in the curriculum of "**Rani Channamma University, Belagavi**".

Guide of the Project
(Laxmi. patil)

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DECLARATION

We hereby declare that the entire work embodied in this project entitled "Linear Transformations and its Matrix" submitted to Department of Mathematics for the partial fulfillment for the award of Bachelor of Science, is the result of investigation carried out by us in the Department of Mathematics under the supervision and guidance of **Miss. Laxmi Patil**,lecturer, Department of Mathematics, S. B. Arts and K. C. P. Science College, Vijayapur. This project has not been submitted for any diploma or degree.

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Write on your own (by Students)

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CHAPTER 1

LINEAR TRASFORMATIONS

1.1 .INTRODUCTION

Linear Transformation is a function from one vector space to another vector space satisfying certain conditions.

In particular linear transformation from R^n toR^m is known as the Euclidean linear transformation .linear transformation have important applications in physics, Engineering and various branches of mathematics.

Definition

Let v and w be two vector spaces

Function$T: V \rightarrow W$ is called a linear transformation from v to w if for all u,u and v and all scalars K,

1.2.Properties of linear transformation

If $T: U \rightarrow V$ is linear transformation, then

01. $T(0) = 0^1$ where o and 0^1 are zero vectors of u and v respectively

$\mathrm{T}(\alpha + 0) = T(\alpha) + T(0)$

$\mathrm{T}(\alpha) = T(\alpha) + T(0)$

$\mathrm{T}(\alpha) + 0^1 = \mathrm{T}(\alpha) + T(0)$

$\mathrm{T}(0)=0^1$

02 consider

$\mathrm{T}[\alpha + (-\alpha)] = T(\alpha) + T(-\alpha)$

$\mathrm{T}(0) = T(\alpha) + T(-\alpha)$

$0^1=\mathrm{T}(\alpha) + T(-\alpha)$

T($-\alpha$) is the inverse of T($-\alpha$)

$$\therefore T(-\alpha) = -T(\alpha)$$

(03) We shall prove the result by mathematical induction

Let p(n):T($c_1\,\alpha_1 + c_2\,\alpha_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots . c_n T(\alpha_n)$

Now

P(1):T ($c_1\alpha_1$)= $c_1 T(\alpha_1)$ as T is linear

As T is linear P(1) is true

Hence the result is true for n=1

Let us assume that p(k) is true for same +ve integer k.

I.e, T($c_1\,\alpha_1 + c_2\alpha_2 \ldots\ldots\ldots\ldots\ldots c_k\alpha_k$)

$$c_1\,T(\alpha_1) + c_2 T(\alpha_2) \ldots\ldots\ldots c_k(\alpha_k) is\ true$$

Now we shall show that p (K+1) is true,

Consider,

T($c_1\,\alpha_1 + \ c_2\,\alpha_2 + \cdots \ldots\ldots + c_k\alpha_k + c_{k+1}, \alpha_{K+1}$) $= 0$

$$T(c_1\,\alpha_1 + c_2\alpha_2 + \cdots \ldots\ldots. + c_k\,\alpha_k\,) + T(c_{K+1} - \alpha_{k+1}) = c_1\,T(\alpha_1) + c_2\,T(\alpha_2)\ldots\ldots. c_k\,T(\alpha_k) + c_{k+1}\,T(\alpha_{k+1})$$

(Since P(K) is true and T is linear

$\therefore P(K+1)$ is true

$\therefore P(K) = P(K+1)$ thus by induction P(n) is true for all integral values of n.

Since the following properties asserts that a linear transformation T can be completely determined by it's values on the elements of a basis of the domain space.

1.2.1.Standard Definition Linear Transformations

Matrix transformation: Let $T: R_n \to R_m$ be a linear transformation then there always exist an m×n matrix A such that

T(X) = AX

This transformation is called the matrix transformation or the Euclidean linear transformation. Hear A is called the standard matrix for T. It is denoted by [T]

For example T: $R_3 \to R_2$ defined by

$T(x, y, z)$ =$(x = y - z, 2y = 3z, 3x + 2y + 5z)$ is a matrix transformation.

Zero Transformation: Let V and W be vector spaces. The mapping $T: \quad V \to W$ *be* defined *by* $T\ (u) \ = 0 \forall\, u \in V$. V s called the zero transformation.

It is easy to verify that T is linear transformation.

Identity Transformation: Let v be any vector space. The mapping $L: V \to V$

Defined by $L\ (u) \ = u \forall\, u \in V$ is called the Identity Transformation on V

It is for the reader to verify that L is linear

1.2.2.Linear Transformation from images of basic vectors

A Linear Transformation is completely defined by the images of any set of basis vectors. Let T: $V \to W$ be a linear transformation and $(v_1, v_2, \dots \dots \dots . v_n)$ can be any basis for v. Then the image $T\ (v)$ of any vector $u \in V$ can be calculated using following steps

Step 01

Express u as a linear combination of the basis vectors $(v_1, v_2, \dots \dots \dots, v_n)$say

$V = k_1 v_1 + k_2 v_2 + \ldots\ldots\ldots + k_n v_n$

Step 02

Apply the linear transformation Ton v as

$T(V) = T(k_1 v_1 + k_2 v_2 + \cdots \ldots\ldots\ldots + k_n v_n)$

$T(v) = k_1 T(v_1) + k_2 T(v_2) + \cdots \ldots\ldots + k_n T v_n$

1.2.3.Composition of Linear Transformation

Let $T_1 : u \to v$ and $T_2 : v \to w$ be a linear transformation. Then the condition of T_2 with T_1 is the linear transformation defined by,

$(T_2\ 0\ T_1)\ (u) = T_2(T_1\ (u))$, where u$\in U$

Suppose that$T_1 : R^n \to R^m$ and $T_2 : R^m \to R^k are$ linear transformation. Then their exist matrices A and B of order m×n and k×m respectively

Such that

$T_1(x)$=A(x) and $T_2(x)$=(Bx)

Thus A =$[T_1]$ and B(T_2)

$((T_2, 0T_1)(x)$=$T_2T_1(x)$ =$T_2(Ax)$=B$(Ax)(BA)x$=$([[T_1][T_2]])(x)$

So we have

$T_2 0 T_1$=$[T_2][T_1]$

Similarly, for 3 such linear transformations

$T_3\ 0\ T_2\ 0\ T_1$=$[T_2]\ [T_1]\ [T_3]$

1.2.4.Theorem 01.Show that the mapping f: $v_3(R) \to v_2(R)$defined by $f\ (a, b, c)\ =\ (c, a\ + b)\ is\ liner$

Proof:→

Let $\propto =(a_1, b_1, c_1)$ and $\beta=(a_2, b_2, c_2)$ be any two elements of $v_3(R)$

Now

$f(\propto +\beta)=f\{(a_1, b_1, c_1) + (a_2, b_2, c_2)\}$

$=f(a_1 + a_2, b_1 + b_2, c_1 + c_2)$

$=\{(c_1 + c_2), (a_1 + a_2) + (b_1 + b_2)\}$

$=\{(c_1, a_1 + b_1) + (c_2, a_2 + b_2)\}$

$=f(a_1, b_1, c_1) + f(a_2, b_2, c_2)$

$=f(\propto) + f(\beta)$

1.2.5.Theorem 02.Show that the mapping $f: v_3(R) \rightarrow v_2(R)$ defined by $f\ (a\ , b\ , c)\ = (a - b, a + \ c)$ *is linear*

Proof:→

Hear $f{:}v_3(R) \rightarrow v_2(R)$ defined as above is said to be linear, if

1. $f(\propto +\beta)=f(\propto) + f(\beta)$

And $f(a \propto) = af(\propto), \forall a \in F, \propto, \beta \in v_3(R)$

Let $\propto= (a_1, a_2, a_3)$ and $\beta = (b_1, b_2, b_3)$ and a$\in F$

Now $f(\propto +\beta)=f\{(a_1, a_2, a_3) + (b_1, b_2, b_3)\}$

$=f(a_{1+}b_1, a_2 + b_2, a_3)$

$=f[(a_1 + b_1) - (a_2 + b_2), (a_1 + b_1) + (a_3 + b_3)]$

$=[(a_1 - a_2), (a_1 + a_3)]+[(b_1 - b_2), (b_1 + b_3)]$

$f(a_1, a_2, a_3) + f(b_1, b_2, b_3) = f(\propto) + f(\beta)$

$\therefore \quad f(\propto +\beta) = f(a)f(\beta)$

2. Also $f(a \propto) = f[a(a_1, a_2, a_3)]$

$=f(aa_1, aa_2, aa_3)$

$=(aa_1 - aa_2, aa_1 + aa_3)$

$=a(a_1 - a_2, a_1 + a_3)$

$=af(a_1, a_2, a_3) = af(\propto)$

DEFINITIONS

RANK OF LINEAR TRANSFORMATION [r(T)]:

Let $T: V \to W$ be a linear transformation the dimension of the range space R (T) is called rank of linear transformation.

RANGE OF LINEAR TRRMATIANSFOON:

Let $T: V \to W$ be a linear transformation then range of T is the set R (T) = $\{T(\alpha); \alpha \in V\}$ .R (T) is the set of all T images of the elements of V therefore R (T) W.

NULLITY OF LINEAR TRANSFORMATION:

Let $T: V \to W$ be a linear transformation the dimension of null space N (T) is called as nullity of linear transformation.

NULL SPACE or KERNEL OF LINEAR TRANSFORMATION:

- Let $T: U \to V$ be a linear transformation .The null space (or kernel) of the subset of U consisting of all vectors u whose image under T is 0 and it is denoted by kernel (T) or N (T).

Elementary operations do not change rank of the matrix.

n$\times n$ identity 1. Rank of the null matrix is not defined=0

2. Rank of matrix =n

3. If A is n$\times n$ non singular matrix then $\rho(A) = \rho(A^{-1})\rho(A) = \rho(A^T) = \rho(AA^T) = \rho(A^{-1}) = \rho(A^0)$

4. $A^0 \to Tranjugate \to Transposet\ conjugate.$

5. $Rank$ Row matrix is $\rho(A) = 0$ or 1

6. $Rank$ Column matrix is $\rho(A) = 0\ or\ 1$

1.3.RANK NULLITY THEOREM:

STATEMENT: Let $T: V \to W$ be a linear transformation and V be finite dimensional vector space then, $r(T) + n(T) = \dim(V)$

OR

Rank + nullity=dimension of domain

PROOF: Let V be a vector space of dimension m

I .e dim V=m and

Dim (N (T)) = n = n (T)

Since nullity of transformation is subspace V.

$$\therefore n < m$$

Let $B_1 = \{\alpha_1, \alpha_2, \dots \dots \dots, \alpha_n\}$ be a basis of nullity of linear transformation N(T).

We will extend this basis to basis of the vector space V.

$$B_2 = \{\alpha_1, \alpha_2, \dots \dots \dots \dots, \alpha_n, \beta_1, \beta_2, \dots \dots \dots \dots, \beta_s\}$$

$n + s = m$

Now, $T(\alpha_1), T(\alpha_2), \dots \dots \dots, T(\alpha_n)$ belongs to null space.

$\therefore T(\alpha_1) = 0, T(\alpha_2) = 0, \dots \dots \dots, T(\alpha_n) = 0$

$$(\alpha_i' s \in N(T))$$

Let $S = \{T(\beta_1), T(\beta_2), \dots \dots \dots, T(\beta_s)\}$

We shall show that the set of S of s vectors of R(T) is a basis of R(T).

1. S spans R(T)

Since, B_2 *is a basis of* V.

$\therefore$ The set $\{T(\alpha_1), T(\alpha_2), \dots \dots \dots, T(\alpha_n)\}$ and

$T(\beta_1), (\beta_2), \dots \dots \dots T(\beta_s)$ Spans of R (T).

But $T(\alpha_1) = 0, T(\alpha_2) = 0, \dots \dots \dots, T(\alpha_n) = 0$

Hence the set S spans R (T7).

2. S is linearly independent

Consider, $C_1 T(\beta_1) + C_2 T(\beta_2) + \cdots + C_s T(\beta_s) = 0$

$T(C_1\beta_{1)} + C_2\beta_2 + \cdots + C_s\beta_s) = 0$

$C_1\beta_1 + C_2\beta_2 + \cdots + C_s\beta_s = 0$

$\therefore$ $\exists$ Scalar $d_1, d_2, \dots \dots \dots, d_n$ such that

$$C_1\beta_1 + C_2\beta_2 + \cdots + C_s\beta_s$$

$$= d_1\alpha_1 + d_2\alpha_2 + \cdots + d_n\alpha_n$$

$\Rightarrow$ $C_1\beta_1 + C_2\beta_2 + \cdots + C_n\beta_n - d_1\alpha_1 - d_2\alpha_2 - \cdots - d_n\alpha_n$

Since B_2 is basis of V

$\therefore$ Vectors in B_2 are linearly independent.

$\therefore$ $d_1 = d_2 = \cdots = d_n = C_1 = C_2 = \cdots = C_s = 0$

S is linearly independent

S is a basis of R (T)

Dim R (T) =S, dim n (T) = n

$$n + s = m$$

$\therefore$ $n(T)$+r (T) =dim V

1.3.1.THEOREM .Let $T: U \rightarrow V$ be a linear transformation then range of T I.E R (T) is subspace of T

Proof →

Since T (0) = 0,s '0' is in R (T) and thus R (T) $\neq$ 0

Let$v_1, v_2 \in R(T)$, Then $v_1 = T(u_1)$ and $T(u_2)$for some u_1 *and* u_2 in U

NOW for a, b∈F $au_1 + au_2 = aT(u_1) + aT(u_2)$

$$= T(au_1 + bu_2) \in R(T)$$

Since $au_1 + bu_2 \in U$. Therefore R (T) is subspace of V

1.4.MATRIX OF LINEAR TRANSFORMATION

The matrix of a linear transformation is a matrix for which $T(\overrightarrow{x}) = A\vec{x}$,for a vector $\vec{x}$ in the domain of T. this means that applying the transformation T to a vector is the same as multiplying by this matrix

Such a matrix can be found for any linear transformation T from R^n to R^m, for fixed value of n and m, and is unique to the transformation. in this lesson, we will focus on how exactly to find that matrix A, called the standard matrix for the transformation.

DEFINITION

Let $T: R_n \rightarrow R_m$ be a linear transformation. Then we can find a matrix such that $T(\overrightarrow{x}) = A\vec{x}$ .In this case, we say that T is determined or induced by the matrix A is known as matrix of linear transformation.

1.4.1.PROPERTIES:HOW TO FIND THE MATRIX OF A LINEAR TRANSFORMATION

In order to find this matrix, we must first define a special set of vector from the domain called the standard basis. The big concept of a basis will be discussed when we look at general vector spaces, for now We just need to understand what vectors make up this set.

The standard basis for R^2 is:

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \overrightarrow{e_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The standard basis for R^3 is:

$$\overrightarrow{e_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \overrightarrow{e_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\overrightarrow{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

See the Pattern? We can define the standard basis like this for any R^n

The standard matrix of a transformation $T: R^n \to R^m$ has Column $T(\overrightarrow{e_1}), T(\overrightarrow{e_2}), \ldots\ldots T(\overrightarrow{e_n})$, where $\overrightarrow{e_1}, \overrightarrow{e_2}, \ldots\ldots \overrightarrow{e_n}$, represent the standard basis that is:

$$T(\overrightarrow{x}) = A\vec{x} \leftrightarrow A = [T(\overrightarrow{e_1}) \; T(\overrightarrow{e_2})]$$

Therefore, to find the standard matrix, we will find the image of each standard basis vectors. This is show in the following examples.

1.4.2.EXAMPLES

1: FIND THE STANDARD MATRIX FOR THE TRANSFORMATION T WHERE:

$$T\left(\begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}\right) = \begin{bmatrix} x1 - x2 \\ 2x3 \end{bmatrix}$$

Solution:

T takes vectors with three entries to vectors with two entries therefore:

$$T: R^3 \to R^2$$

So the domain of T is R^3.to find column of the standard matrix for the transformation we will need to find:

$$T(\overrightarrow{e_1}).T(\overrightarrow{e_2}), \; and \; T(\overrightarrow{e_3})$$

Using the given rule for T:

$$T(\overrightarrow{e_1}) = T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)$$

$$= \begin{bmatrix}1-0\\2(0)\end{bmatrix}$$

$$= \begin{bmatrix}1\\0\end{bmatrix}$$

1.4.3. FIND THE STANDARD MATRIX FOR THE TRANSFORMATION T WHERE:

$$\boldsymbol{T\left(\begin{bmatrix}x1\\x2\\x3\end{bmatrix}\right) = \begin{bmatrix}x1-x2\\2x3\end{bmatrix}}$$

Solution:

T takes vectors with three entries to vectors with two entries therefore:

$$T: R^3 \to R^2$$

So the domain of T is R^3.to find column of the standard matrix for the transformation we will need to find:

$$T(\overrightarrow{e_1}).T(\overrightarrow{e_2}), \ and \ T(\overrightarrow{e_3})$$

Using the given rule for T:

$$T(\overrightarrow{e_1}) = T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)$$

$$= \begin{bmatrix}1-0\\2(0)\end{bmatrix}$$

$$= \begin{bmatrix}1\\0\end{bmatrix}$$

1.4.4.THEOREM 01:EVERY MATRIX TRANSFORMATION IS A LINEAR TRANSFORMATION

Suppose that T is a matrix transformation such that $T(\overrightarrow{x}) = A(\overrightarrow{x})$ for some matrix (A) and that the vectors $\overrightarrow{u}$ and $\vec{v}$ are in the domain. Then for arbitrary scalars c and d:

$$T(c\vec{u} + d\vec{v}) = A(c\vec{u} + d\overrightarrow{v})$$

$$= A(c\vec{u}) + A(d\vec{v})$$

$$= cA\vec{u} + dA\vec{v}$$

$$= cT(\vec{u}) + dT(\vec{v})$$

$$T(c\vec{u} + d\vec{v}) = cT(\vec{u}) + dT(\vec{v})$$

T must be a linear transformation

1.5.APPLICATIONS

Matrix of linear transformation can be used to merge multiple transformations together into a single transformation

Linear transformation have numerous applications is various fields, including:

1. Linear Algebra: They are used to solve systems of linear equations, find eigenvalues and eigenvectors, and diagonalize matrices.
2. Computer Graphics: Linear transformations are used to perform rotations, scaling, and
3. Machine Learning: They are used in neural networks, principal component analysis (PCA), and singular value decomposition (SVD).
4. Physics and Engineering: Linear transformations describe the motion of the objects, including rotations, reflections, and projections.

5. Data Analysis: they are used in data compression, image

6. Differential Equations: Linear transformations help solve differential equations and analyze stability.

7. Markov Chains: They are used to model random processes and calculate steady-state probabilities.

8. Optimization: Linear transformations are used in Linea programming and optimization techniques.

9. signal Processing: They are used in filtering, convolution, and Fourier analysis.

10. Statistics: Linear transformations are used in filtering, hypothesis testing, confidence intervals, and regression analysis.

These applications leverage the power of linear transformations to simplify complex problems, reveal underlying structures, and facilitate efficient computations.

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CONCLUSION

A linear transformation is a mathematical function that maps vector from one vector space to another while preserving the operations of vector addition and scalar multiplication. The key conclusions about linear transformation are:

1.**Preservation of operations:** Linear transformations preserve the operations of vector addition and scalar multiplications

2. **Matrix representation:** Every linear transformation can be represented by a matrix.

3. **Composition:** The composition of two linear transformations is also a linear transformation.

4. **Inverse:** If a linear transformation is invertible, its inverse is also a linear transformation.

5. **Kernel and image:** Every linear transformation has a kernel (null space) and an image (range).

6. **Rank-nullity theorem:** The rank of a linear transformation plus the nullity equals the dimension of the domain.

7. **Invertability:** A linear transformation is invertible if and only if its matrix representation is invertible.

These conclusion from the foundation of linear algebra and are crucial for understanding many mathematical and real-world applications, such as data analysis, machine learning, and physics.